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Free vibration analysis of circular and annular thin plates based on crack characteristics

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ABSTRACT

This study presents the effect of vertical and horizontal oriented cracks on free vibration response for circular and annular thin plates. To investigate the dynamic behavior of the damaged circular structures the cracks are modeled separately considering horizontal/vertical orientations, ten different locations, and four crack sizes. For annular thin plates, vertical and horizontal oriented cracks are placed in the middle between the outer and inner edges to investigate the effect of crack directions. The first five resonant frequencies, and the corresponding mode shapes of the cracked circular and annular plates, are obtained by employing the Finite Element Method. The free vibration analyses are conducted considering clamped boundary conditions for circular plates and clamped-clamped, clamped-free, and free-clamped boundary conditions for annular plates. The results are presented and interpreted considering the differences in the non-dimensional frequencies and the mode shapes of those structures. According to the findings, it is seen that depending on its location and size, a crack can change the mode shapes by accumulating the bending regions around it. Besides, a crack may also change the number of bending regions that occurred in the mode shapes of the circular or annular structures.

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1. Introduction

Engineering structures are designed in such a way that they have to endure to subjected static and dynamic loads. These structures may adversely be affected by all kinds of damage that may occur due to any reason. Particularly, structural elements may reveal different behaviors due to their design under dynamic excitation. For this reason, understanding the effects of possible damages on engineering structures is of great importance in terms of both determining the location of the damage and being prepared for the consequences that the damage may reveal. Circular and annular geometries are also frequently used in engineering structures for both functionalites (e.g., design requirements, ease of assembly, stress distribution, ventilating, etc.) and aesthetic purposes. In the literature, circular, annular, and circular cross-sectioned thin plates have been examined considering various boundary conditions. Some of those works are briefly presented as follows. Huang and Ma (2000) investigated the vibration analysis of the side-cracked circular plates. Chakraverty et al. (2001) used orthogonal polynomials as shape functions for the vibration response of elliptic plates. Zhou et al. (2011) employed the Hamiltonian approach for the vibration response of circular and annular thin plates. Shi et al. (2014) proposed a simple solution for free vibration analysis of circular, annular, and sector plates. Khare and Mittal (2015) performed the numerical analysis of a thin circular plate with general boundary conditions.

Arshad et al. (2016) studied the vibration response of sandwich cylindrical shells. Xie et al. (2017) studied the wave-based method for free and forced vibration analysis of annular plate and cylindrical shell plates. Jaiman and Singh (2019) performed a numerical simulation of the free vibration response of circular and annular plates. Zhang et al. (2019) used a simplified plate theory for vibration response of annular, sector, and circular plates. Bisheh et al. (2019) examined the static and dynamic analysis of circular and annular plates using the differential quadrature method. Cuong-Le et al. (2021) and He et al. (2021) investigated conical, cylindrical, and annular plates in terms of different materials and boundaries. Sun et al. (2021) investigated the higher-order free vibration analysis with multiple cutouts on elastic plates. On the other hand, Gonenli et al. (2021), and Gonenli and Das (2021) investigated the effect of crack on vibration response and dynamic stability for the different types of plate structures.

In this paper, the effect of damage on circular and annular plates with different boundary conditions is investigated. After the present healthy model has been validated, a singular crack is added to the structure. Free vibration analyses are carried out on fully clamped circular plates using the finite element method at ten different locations for each crack. Each crack can be in vertical or horizontal directions with three different lengths for the fully clamped circular plates. In addition, annular plates are examined under three different boundary conditions, which are fully clamped, clamped from the outer circle, and clamped from the inner circle. Cracks are also oriented in vertical or horizontal directions. The first five resonant frequencies and corresponding mode shapes of the circular plates and annular plates are obtained. The cracked structures and healthy structures are compared in terms of free vibration responses and mode shapes. It is concluded that depending on its location and size, a crack can change the mode shapes by accumulating the bending regions around it. Besides, a crack may also change the number of bending regions that occurred in the mode shapes of the circular or annular structures.

2. Material and Method

Two different structures, circular and annular plates, are examined within the scope of the research. The impact of crack direction, location, and length on the free vibration behavior of a circular thin plate is investigated with a fully clamped boundary (C) condition. For the annular thin plate, the crack with the highest size, which is considered for the circular plate, is taken into account. All cracks are considered in vertical and horizontal directions. For the annular plate, three boundary conditions namely, fully clamped (CC), outer clamped – inner free (CF), and outer free – inner clamped (FC). Representative circular and annular plates are given in Figure 1. Here, l represents the distance from the center of the horizontally and vertically positioned crack, *c* represents the length of the crack, R_1 is the outer diameter, and R_2 is the inner diameter.

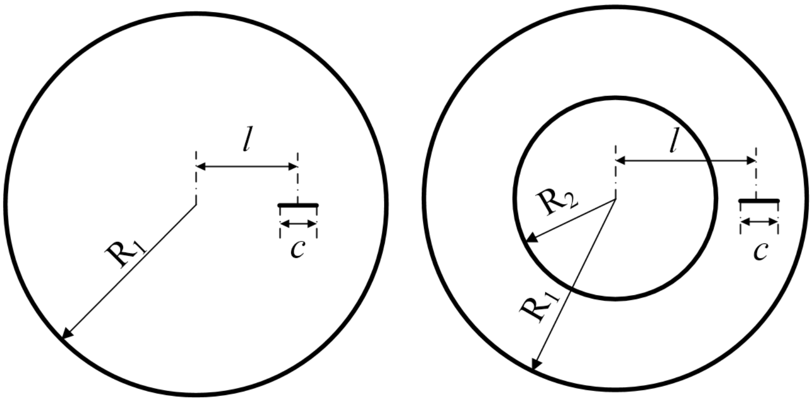

Figure 1. Circular and annular plates with crack.

As material specifications, the following properties are considered. Young's modulus (E) is 2.06 x 10^{11} Pa, Poisson's ratio (v) is 0.3, and density (ρ) is 7850 kg/m^3. h/R_2 is taken as 0.01, and the ratio of R_2/R_1 is taken as 0.4 (Shi et al. 2014 & Zhou et al. 2011). The flow chart for ANSYS analysis is shown in Figure 2.

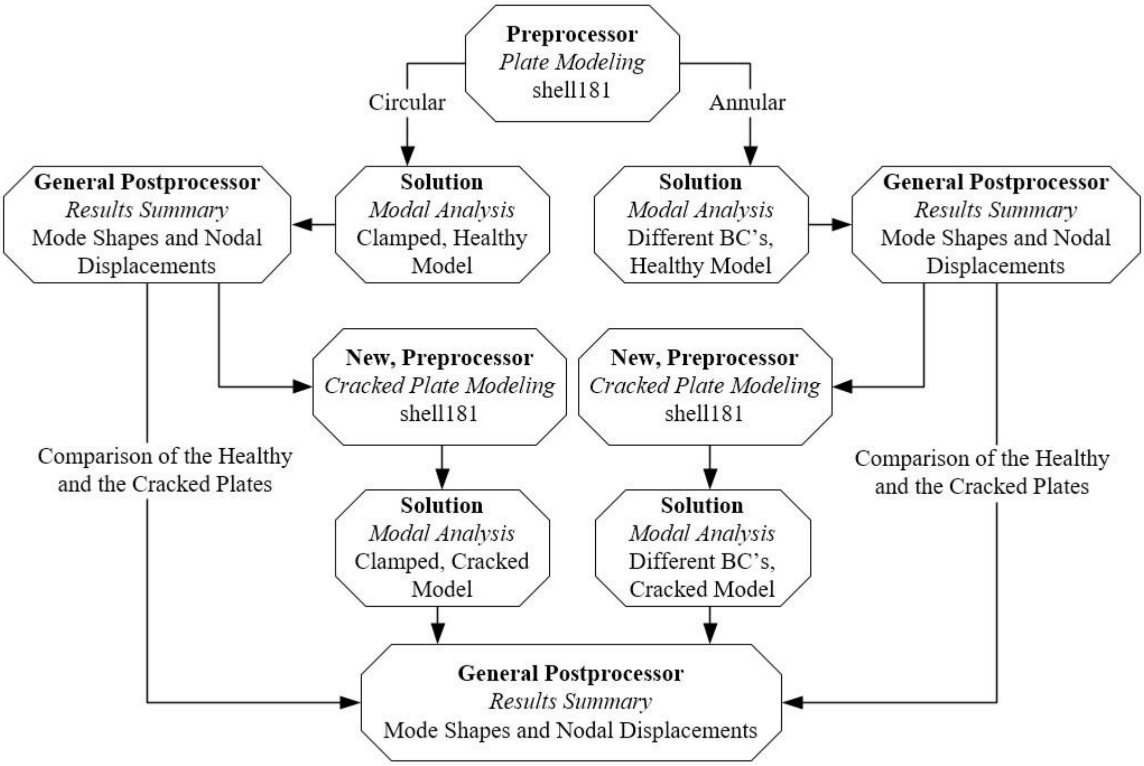

Figure 2. The flowchart of the ANSYS procedure.

The effect of the crack on five different mode shapes and five different natural frequencies of annular and circular plates under three different boundary conditions were investigated for both crack directions. For the annular plate, the center of the crack is located at l/R_1 ratio equal to 0.7. For the circular plate, free vibration analysis is carried out by implementing the crack separately to ten different locations, both horizontally and vertically, with an l/R_1 ratio varying between 0 (Loc 1) and 0.9 (Loc 10) with an interval of 0.1. In addition, each analysis is repeated for three different crack lengths to examine the size effect. For this purpose, the cracks are considered with lengths of $0.05R_2$ (Size 1), $0.10R_2$ (Size 2), and $0.15R_2$ (Size 3), respectively. All cracks have the same width of $0.005R_2$. All obtained data are presented and interpreted in terms of dimensionless frequency parameters and frequency ratios. Eq. (1) gives the dimensionless frequency parameter, λ.

$$\lambda = \omega R_1 \sqrt{\rho h / D} \tag{1}$$

Here, ω represents the natural frequency. D is flexural rigidity and given in Eq. (2).

$$D = \frac{Eh^3}{12(1-\nu^2)} \tag{2}$$

3. Numerical Results

This study measures the impact of the crack on the first five natural frequencies and corresponding mode shapes of the circular and annular plate structures. Before proceeding the free vibration analysis, the validation of the considered finite element model has been conducted.

To validate the present model, various studies in the literature are considered. The validation analysis results of the clamped circular thin plate are given in Table 1. In Table 2, the validation analysis results of the annular thin plate with three different boundary conditions, outer - inner circles clamped (CC), outer circle clamped – inner circle free (CF), and outer circle free – inner circle clamped (FC), are given, respectively. The comparison of the nondimensional frequency parameters confirms the accuracy of the SHELL181 element type and the mesh density used in ANSYS.

Table 1. Dimensionless frequency parameters for the clamped circular plate.

Study	Vibration Mode				
	1	2	3	4	5
Leissa (1969)	10.22	21.26	34.88	39.77	51.03
Blevins (1993)	10.22	21.26	34.88	39.77	51.04
Han and Liew (1999)	10.23	21.28	34.89	39.79	51.03
Liew and Yang (1999)	10.25	21.33	34.97	39.88	51.16
Zhou et al. (2011)	10.22	21.26	34.88	39.77	51.03
Shi et al. (2014)	10.21	21.26	34.87	39.76	51.02
Present Study	10.21	21.25	34.87	39.76	51.01

Table 2. Frequency parameters for the annular plate with different boundary conditions.

Case / Study		Vibration Mode				
		1	2	3	4	5
Fully Clamped (CC)	Chakraverty (2001)	61.88	63.04	66.87	74.97	86.88
	Zhou (2011)	61.87	63.00	66.67	73.63	84.59
	Present Study	61.81	62.93	66.59	73.52	84.45
Clamped (Outer Edge - CF)	Chakraverty (2001)	13.50	19.46	31.74	47.81	66.81
	Zhou et al. (2011)	13.50	19.39	31.34	46.86	65.98
	Present Study	13.60	19.55	31.48	46.92	65.96
Clamped (Inner Edge - FC)	Zhou (2011)	9.02	9.11	10.45	14.94	22.95
	Present Study	9.07	9.13	10.37	14.73	22.53
	Chakraverty (2001)	9.08	9.18	10.53	15.34	23.33

Figures 3 and 4 present the first five natural frequency variations for the fully clamped circular plates, considering ten different crack positions, crack direction, and crack size. The frequency ratio ($\omega_{cracked} / \omega_{healthy}$) values close to 1 indicates the regions where the crack does not affect the plate's resonant frequencies. This ratio decreases in regions where that effect is high. The impact of the crack on the natural frequency values of the circular and annular plates becomes more visible as its size grows. However, such an impact is also depended on the location of the crack since at certain locations the crack does not significantly impact the natural frequencies even if its size is increased. On the other hand, this is not a general conclusion since a larger crack may affect the natural frequency of the structure especially if it intercepts the bending regions of the mode shapes of the structure. The "location-frequency ratio" curve shows different behavior in different mode shapes depending on the crack's location. Besides, the direction of the crack creates different effects on the structure. Due to the geometric form of the relevant mode shape, each location-frequency ratio curve is different from each other due to the cracks that are occurred in the bending regions of the mode shapes. However, the variation of the crack size does not change the critical bending regions of the same mode shape of the structure.

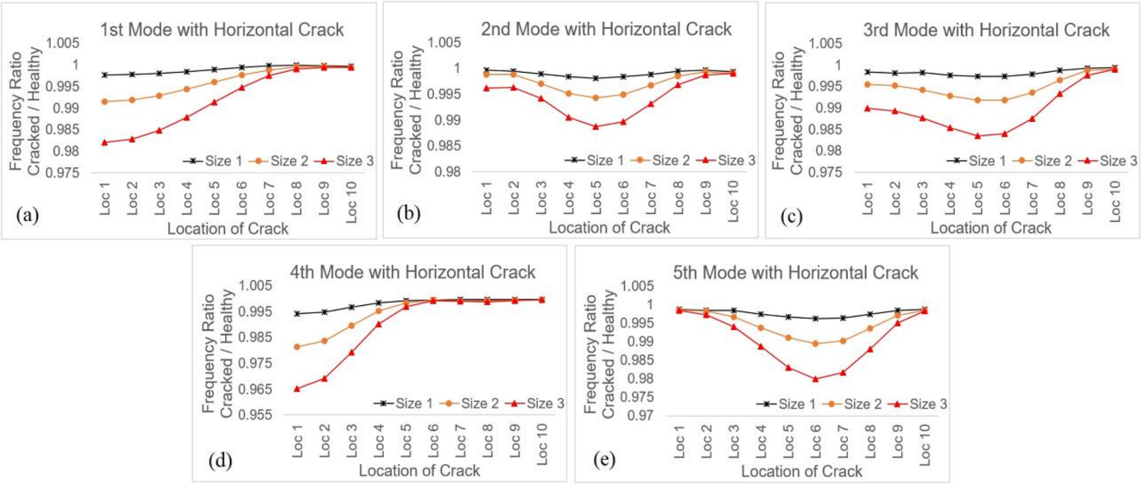

Figure 3. Location-frequency ratio curves considering the effect of the horizontal crack

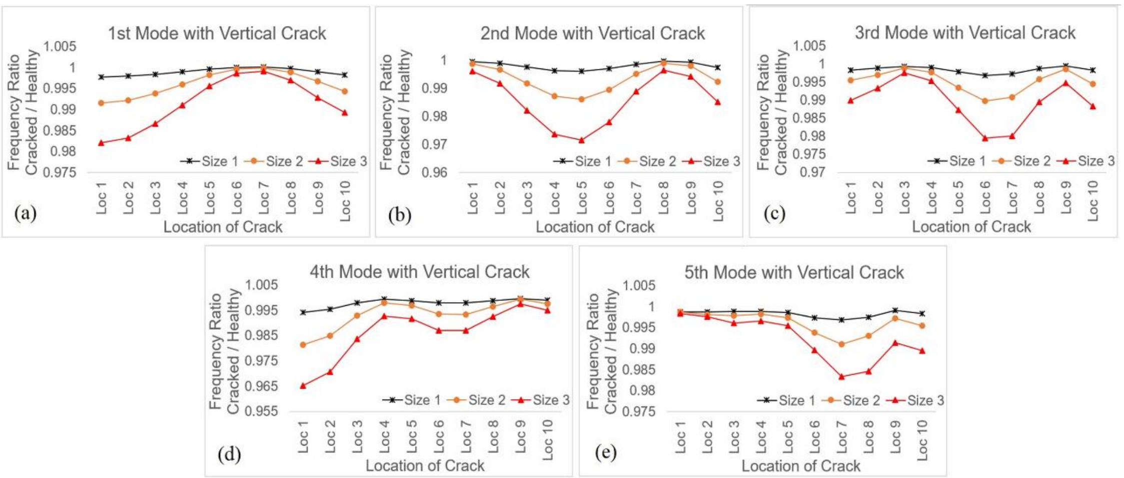

Figure 4. Location-frequency ratio curves considering the effect of the vertical crack

Considering the differences in the variation of the curves shown in Figures 3 and 4, it is understood that the differences in the location-frequency ratio curve take place due to the corresponding mode shapes. In Figure 5, the first five mode shapes for the clamped healthy circular structure and those of the most-affected cracked cases are presented together. The presence of the crack in the structure changes the relevant mode shape characteristics. However, it also increases the displacement amplitude of the vibration. The vibration pattern of the healthy mode shape changes into an unsymmetrical pattern due to the location of the crack.

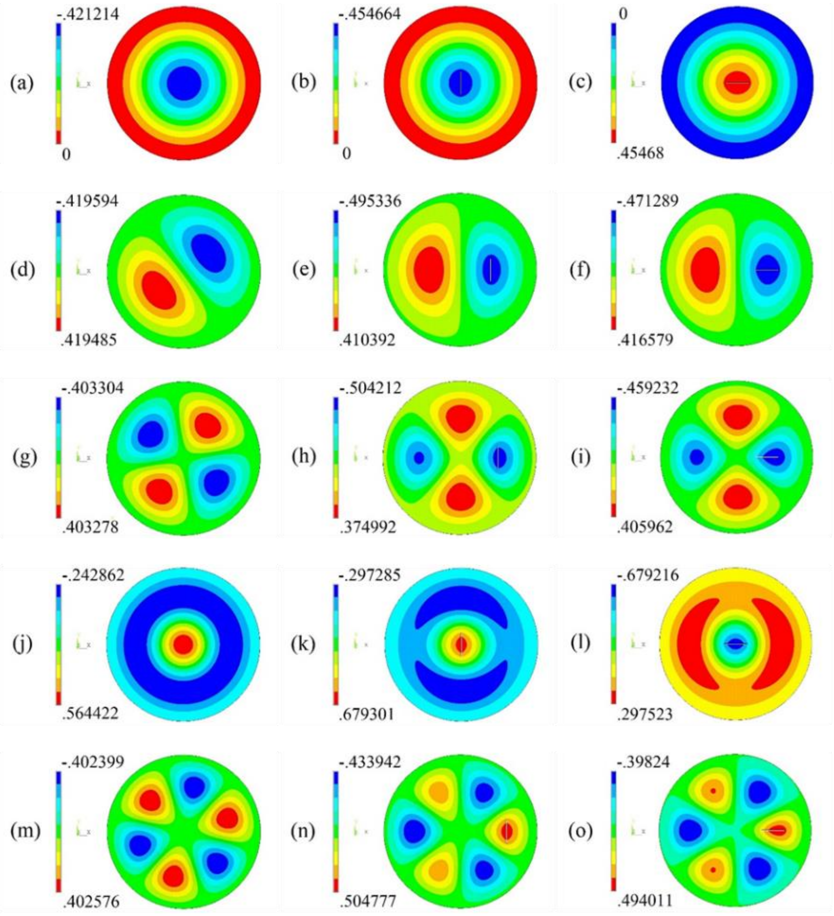

Figure 5. The first five mode shapes of the clamped healthy circular plate (a, d, g, j, m), vertical-cracked circular plate (b, e, h, k, n), and horizontal-cracked circular plate (c, f, i, l, o).

In Figure 6, the first five mode shapes of the healthy, the vertical-cracked, and the horizontal-cracked annular plate under CC boundary conditions are presented.

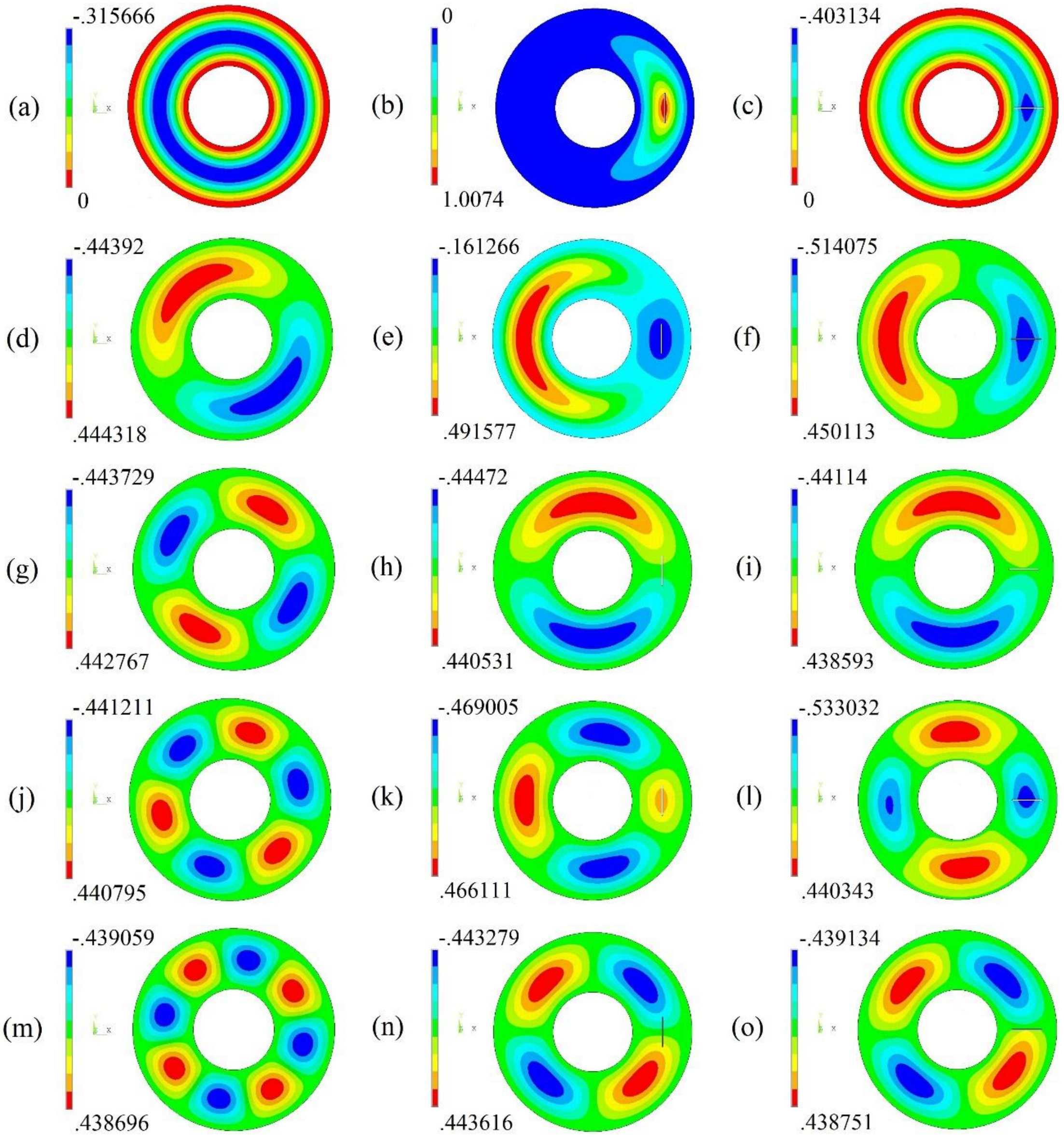

Figure 6. The first five mode shapes of the CC healthy circular plate (a, d, g, j, m), vertical-cracked circular plate (b, e, h, k, n), and horizontal-cracked circular plate (c, f, i, l, o).

It is seen that the vertical and horizontal cracks affect the mode shape in different ways. While the mode shape symmetry of the healthy plate disappears in cracked ones, the vibration amplitude increases and changes towards the cracked side. Especially in the first mode, the mode shape completely changes and a new mode shape occurs with the presence of the crack. In the second and the third modes, it is seen that the displacement amplitude becomes wider in the opposite position of the crack location. As seen from Figure 6, the number of bending regions for the third, fourth, and fitfth mode shapes is decreased with the presence of the crack no matter it is horizontal or vertical.

Figure 7 presents the first five mode shapes of the healthy, horizontally cracked, and vertically cracked annular plates under CF boundary conditions. The first three mode shapes of the CF annular plate are mostly similar to the mode shapes of the healthy plate. However, it is seen that the symmetry in the form of the second and third mode shapes shifts towards the damage direction of the presence of the crack. Vibration amplitude also increases in these cases. A similar impact that is observed for the CC annular structure is seen for the fourth and fifth mode shapes of the cracked CF annular structure.

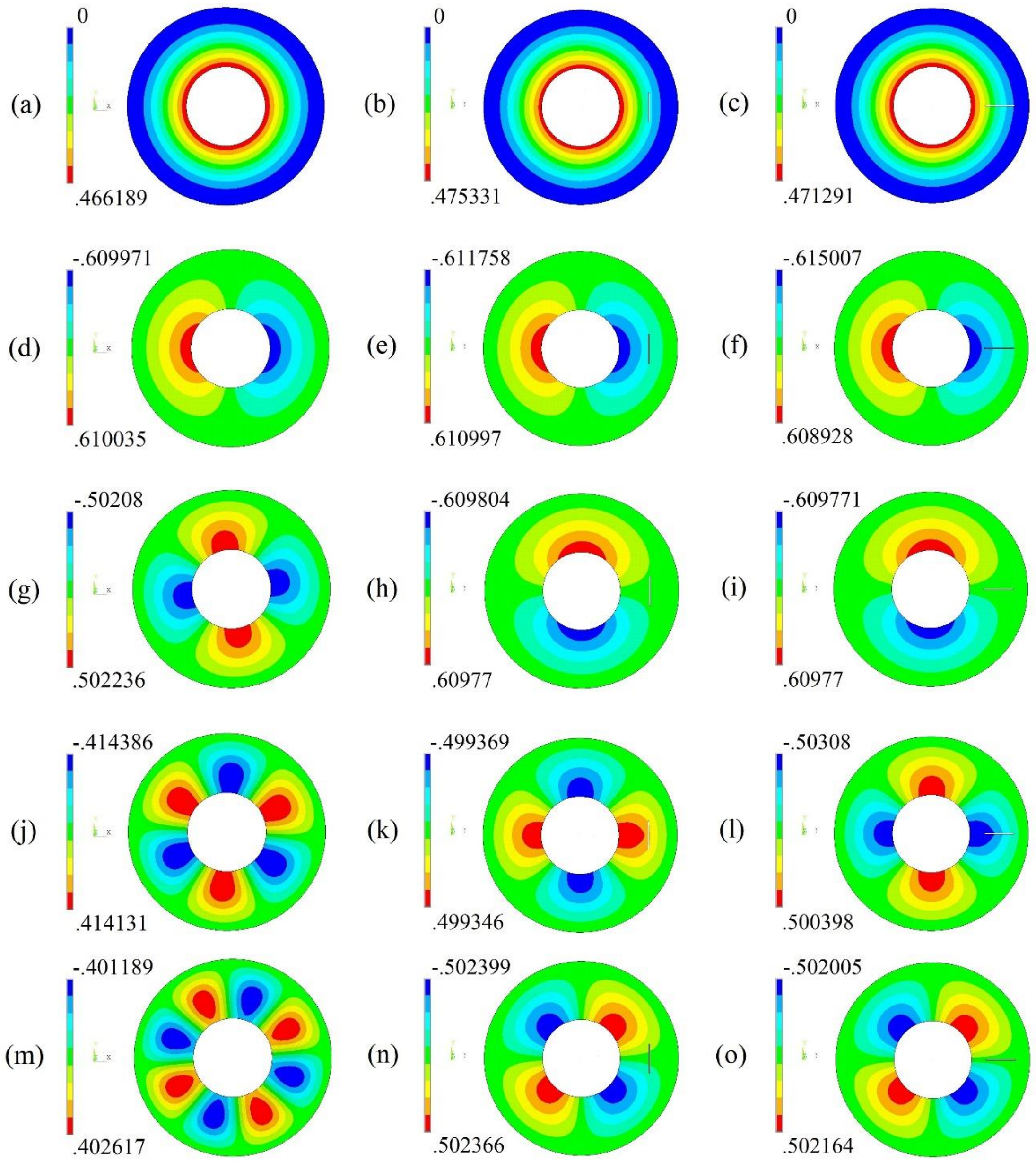

Figure 7. The first five mode shapes of the CF healthy circular plate (a, d, g, j, m), vertical-cracked circular plate (b, e, h, k, n), and horizontal-cracked circular plate (c, f, i, l, o).

The first five mode shapes of the healthy, horizontally cracked, and vertically cracked plates under FC boundary conditions are shown in Figure 8. The crack affects the symmetry in the first and the second mode shapes and changes the maximum bending region of the first mode shape from the outwards of the structure to its center. The crack affects the third mode in such a way that it merges the conjugated bending regions and changes their occurrence zones. In the presence of horizontal or vertical crack, the fourth and fifth mode shapes shifted to the third mode shape of the healthy structure.

The direction of the crack affects also the natural frequencies of the annular plates with three different boundary conditions. Tables 3-5 shows the effect of the crack direction on the first five natural frequencies of the annular plates. It is seen from Tables 3-5 that the change in the natural frequency values is slight even for the highest crack size. However, as seen from Figures 6-8, they considerably impact the mode shapes of the structures by changing their symmetric pattern, numbers of bending regions, or maximum bending region locations. Since the highest crack size is considered for the annular structures, it can be also interpreted that the smaller cracks would not affect the structure differently. As they are examined for the circular structure, the difference only takes place for the amplitude of the natural frequencies.

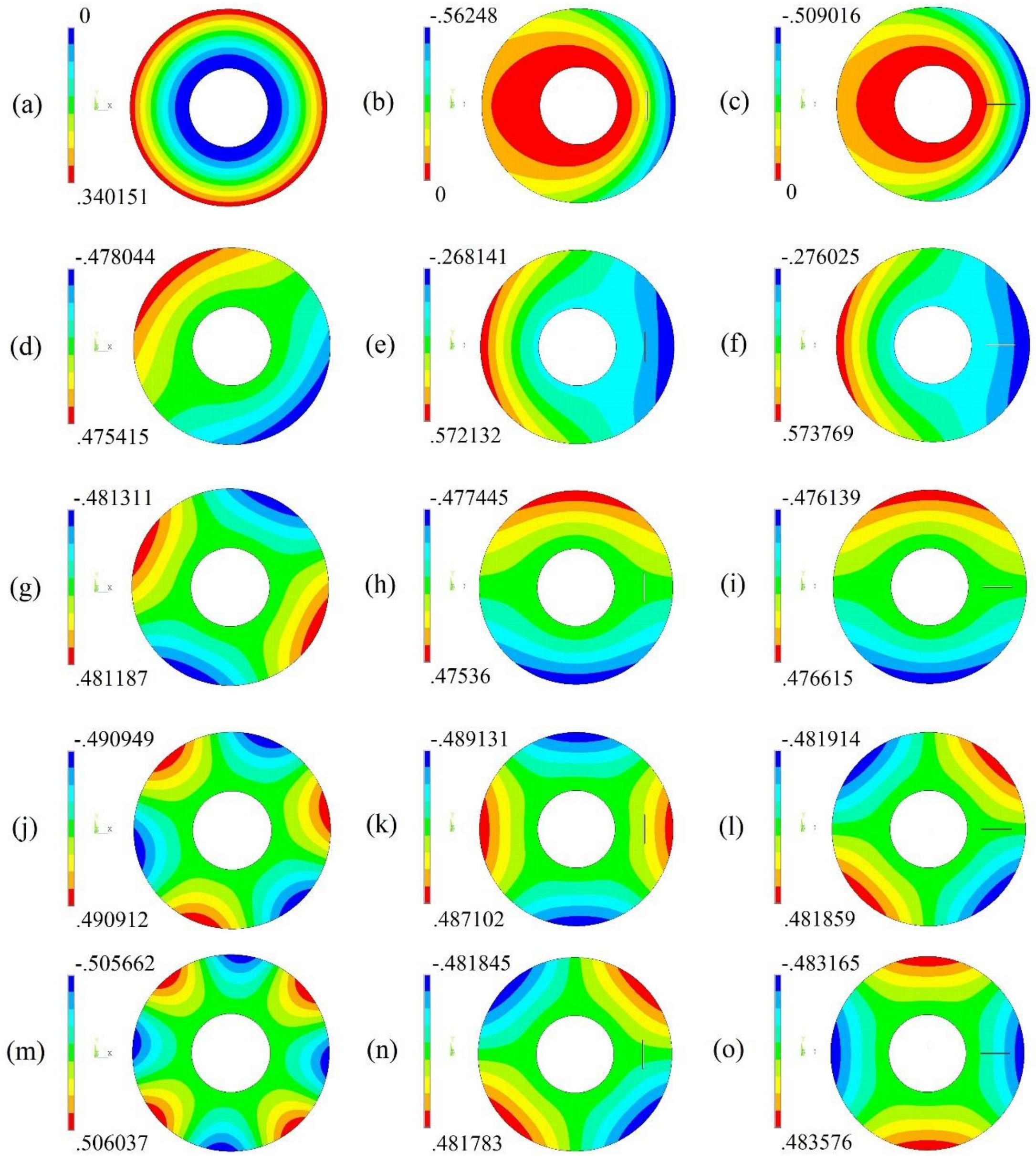

Figure 8. The first five mode shapes of the FC healthy circular plate (a, d, g, j, m), vertical-cracked circular plate (b, e, h, k, n), and horizontal-cracked circular plate (c, f, i, l, o).

Table 3. Comparison of the frequency parameters of the healthy and the cracked annular plate under CC boundary conditions

Case		Vibration Mode				
		1	2	3	4	5
Fully Clamped (CC)	Healthy	61.8105	62.9271	66.5851	73.5180	84.4534
	Vertical Crack	60.6776	62.3496	65.8981	72.8067	83.7482
	Horizontal Crack	61.7862	62.8643	66.4412	73.2059	83.8738

Table 4. Comparison of the frequency parameters of the healthy and the cracked annular plate under CF boundary conditions

Case		Vibration Mode				
		1	2	3	4	5
Clamped (Outer Edge - CF)	Healthy	13.5981	19.5461	31.4848	46.9152	65.9650
	Vertical Crack	13.5756	19.5376	31.4180	46.6781	65.0976
	Horizontal Crack	13.5926	19.5214	31.3916	46.6578	65.4117

Table 5. Comparison of the frequency parameters of the healthy and the cracked annular plate under CC boundary conditions

Case		Vibration Mode				
		1	2	3	4	5
Clamped (Inner Edge - FC)	Healthy	9.0185	9.1111	10.4531	14.9421	22.9469
	Cracked, Vertical	8.9952	9.0880	10.4383	14.9320	22.9347
	Cracked, Horizontal	8.9883	9.0906	10.4460	14.9219	22.8638

4. Conclusions

In this study, the effect of the crack in different sizes, directions, and sizes, on free vibration responses and mode shapes of the structure, are investigated. In this context, free vibration analysis of two different structures, circular and annular, is carried out with the finite element method. According to the results, it has been observed that the characteristics of the crack directly affect the dynamic behavior of the structure, and the structure gains new dynamic characteristics. The generalized results can be summarized as:

- The presence of the crack changes the structure’s mode shape in the circular and annular plates. Depending on the crack position, a crack may cause a new mode shape, a shift in the mode shape, a change in the maximum bending region location, and an unsymmetrical distribution of bending regions where they are formed around the crack.
- The location-frequency ratio curve has been mostly affected by the cracks which change the mode shapes of the structure.
- When the crack location coincides with the bending region of the relevant mode-shape and the cracked region is forced to deform, the resonant frequencies are affected more than those of other crack cases.
- According to the relevant mode shapes of the cracked structures, if the location of the crack is in the neutral (no-displacement) region, the frequency is not affected since the crack is not forced to be deformed.
- The increase in the size of the crack affects the frequency more if the damage is located in the bending region of the mode shape of the cracked structure.
- The increase in the size of the damage remains negligible if the damage is in the neutral zone of the mode shape of the cracked structure.
- The direction of the crack is a parameter that affects the mode shape and frequency ratio change for all the plates just like its location and size.

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