

Optimization of two-speed planetary gearbox with brakes on single shafts

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ABSTRACT

This paper deals with configurations of complex planetary gear trains consisting of two planetary gear trains of basic type. These gear trains are formed by linking shafts from different component gearsets and contain two carriers, and therefore designated as two-carrier planetary gear trains with four external and two coupled shafts. The structural configurations are pointed out, and additional research has been made into gear trains using coupled external shafts for torque input and output, with the controlling brakes acting on single external shafts. The kinematic schemes have been created for all analyzed PGT variants, and the available transmission ratio ranges calculated for both speeds. The transmission ratio is changed by alternating the activation of each brake, enabling their use as transmissions with two transmission ratios in transportation technology and other engineering applications. Extreme transmission ratio changes have been determined for each analyzed PGT design solution. Also, relations of ideal torque ratios to the required transmission ratios of component planetary gear trains for both speeds have been calculated. These relations enable the selection of compound gear train designs which will achieve the required pair of transmission ratios. The optimal design parameters for the adopted configuration were determined, and the optimal transmission solution for the given input data selected.

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1. Introduction

There are many benefits that are inherent to planetary gear trains (PGTs) which make them more suitable than classical gear trains. The most important of these advantages is a considerable reduction of mass and dimensions for the same torque rating. Because of that, the application of PGTs has been significantly expanded in various engineering applications.

PGTs as a totality, and particularly complex multi-carrier PGTs cover a vast area of technical knowledge (Arnaudov & Karaivanov, 2005). By connecting the shafts of various gear train units, compound multi-carrier PGTs can be built. PGTs with two coupled shafts and four external shafts are a particular kind of complex multi-carrier PGTs which enable two-speeds. These PGTs have many advantages. The most frequently mentioned is the potential for transmission ratio changing under load. This presents a significant advantage in their application and might be even required in some cases.

There has been no systematic research into these PGTs until now. The review of 15 reversible transmission configurations with two copuled and four external shafts has been presented in (Kudriavtsev & Kirdiyashev, 1977), and rough transmission ratio values as well as rough efficiency values have been provided.

Some scattered reviews of chosen configurations have been performed in (Merritt, 1947; Lechner & Naunheimer, 1999; Arnaudov et al., 2005; Jelaska, 2012), while (Karaivanov & Troha, 2006) indicates the properties of some configurations, however the optimal configuration methodology has not been included. The torque method established by Arnaudov is introduced as the principal tool of systematic analysis of multi-carrier PGTs in (Arnaudov & Karaivanov, 2013; Arnaudov & Karaivanov, 2001; Arnaudov & Karaivanov, 2012). This method may be considered universal and contributes to clarity, as it is applicable not only to two-carrier PGTs, but also to multi-carrier PGTs, as pointed out by the authors in (Arnaudov & Karaivanov, 2001).

The procedure for selection of optimal PGT compound structures has been included in the software DVOBRZ and is based on suggestive systematic research delivered between 2006 and 2011 (Troha, 2011). All the achievable configurations of these transmissions and their working regimes from the software DVOBRZ have been researched by Troha in (Troha et al., 2012) and (Troha et al., 2013). The selection process of a two-speed PGT considered for use as a fishing boat transmission has been discussed in (Stefanović-Marinović et al., 2017).

The selection of an optimal transmission of this type which can satisfy specific requirements is complex, and it can be performed by means of multi-criteria optimization. The usage of multi-criteria optimization to gear trains, and particularly planetary gear trains, has not been the topic of many studies, however an overview can be given. The usage of multi-purpose optimization access, predicated on the Pareto optimality concept, to helical gears design was proposed in (Tudose et al., 2008), while the choice of the best optimization parameters for getting the necessary gear quality and the optimization of the design procedure itself was provided in (Tkachev & Goldfarb, 2009).

Planetary gear transmissions have been the subject of studies appropriate to the optimizations argued in this paper. A comprehensive method for researching the transmission ratio, the internal power flows and the efficiency of complex multi-planetary gear trains was covered in (Arnaudov & Karaivanov, 2005), while the determination of an optimised planetary gear train with a short center distance, light weight and high mechanical efficiency as an important issue in the preliminary design of power transmissions is presented in (Daoudi et al., 2019), where traditional optimisation methods and non conventional methods, such as Genetic Algorithm Optimisation are used to determine the optimal dimensions of epicyclic gear train components. Furthermore, conventional mathematical methods may be combined with methods of artificial intelligence, as in (Čabala & Jadlovska, 2020), where the solution of multi-objective optimization of the production process of an automated assembly line model was presented.

This paper relates to complex planetary gear trains which enable two-speeds by alternative activation of brakes placed on single shafts. The computer program DVOBRZ has been developed for investigation and optimization of two-speed PGTs of these type and potentialities of these software are showed. Also, the computer program PLANGEARs (Stefanović-Marinović, 2011) is used to define the design parameters of the selected variant of two-speed planetary gear box. The described procedure is applied to a numerical example where an optimal transmission defined by structure and design parameters is realized by combining both softwares.

2. Two Speed Compound Planetary Trains

A mechanism obtained by joining two shafts of one PGT unit to two shafts of the other PGT unit is shown in Fig. 1. There are four external shafts, among which two are coupled and two are single external shafts. The whole mechanism is specified as the compound train, while planetary units are specified as component trains.

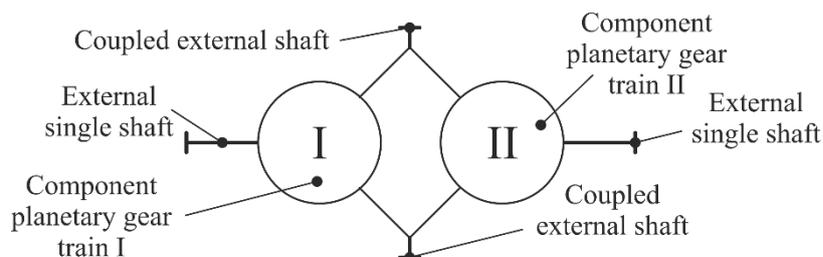


Figure 1. Compound train

Both component trains are planetary gear trains of basic type, i.e., planetary gear trains comprising of a sun gear 1, planet gear 2, ring gear 3 and planet carrier h, as shown in Fig. 2. The ideal torque ratio and element torque ratios, besides the Wolf-Arnaudov symbol for this basic type of PGT are laid out in Fig. 2. As the negative transmission ratio is achieved by locking the carrier, the carrier shaft is the summary element.

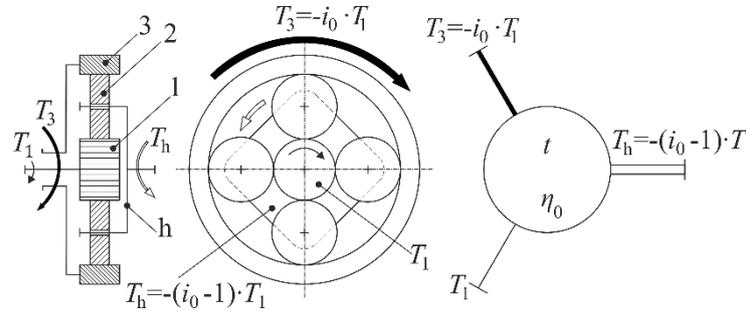


Figure 2. The most used basic type of planetary gear train and the torques acting on its elements (Arnaudov & Karaivanov, 2005)

Additionally, the efficiency of the basic PGT can be ideally defined as:

$$\eta_0 = \eta_{13(h)} = \eta_{31(h)} = 1 \tag{1}$$

In this ideal case the overall efficiency η_0 of the basic PGT with the planet carrier h locked equals unity, and the efficiency of the PGT remains equal to unity regardless of whether the sun gear 1 is driving with planet carrier locked ($\eta_{13(h)}$) or the ring gear 3 is driving with planet carrier locked ($\eta_{31(h)}$). The ideal torque ratio t is defined by means of the number of teeth of gear 1 z_1 and the number of teeth of the ring gear 3 z_3 as:

$$t = \frac{T_3}{T_1} = \left| \frac{z_3}{z_1} \right| = -i_0 > +1 \tag{2}$$

The element torque ratios are given as:

$$T_1 : T_3 : T_h = +1 : (-i_0) : (i_0 - 1) \tag{3}$$

2.1. Structure and Labelling

The structure of compound PGTs is depicted systematically in (Troha et al., 2014; Stefanović-Marinović et al., 2017; Troha et al., 2020, 2020a), however a brief description will be carried out in this paper. There are 12 distinct methods for component train connection in total (Stefanović-Marinović, 2011). An alphanumeric label (S11...S56) is joined to each of those 12 structural schemes, providing an indication of the component train shafts connection models (Fig. 3).

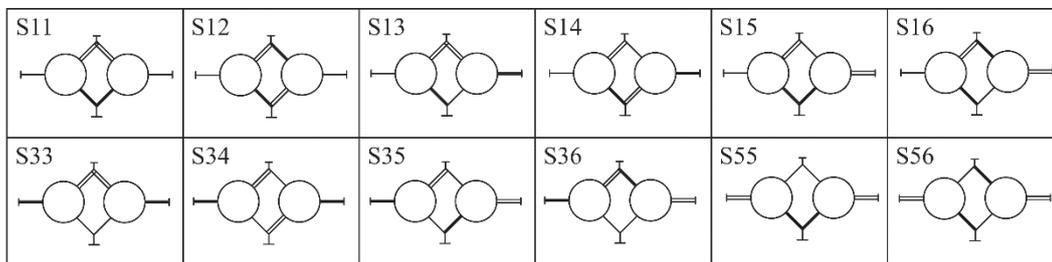


Figure 3. Schemes of planetary gear trains with four external shafts

The brakes can be placed on external shafts, as well as on the input and output shaft, defining the layout variants (label V1...V12), shown in Fig. 4.

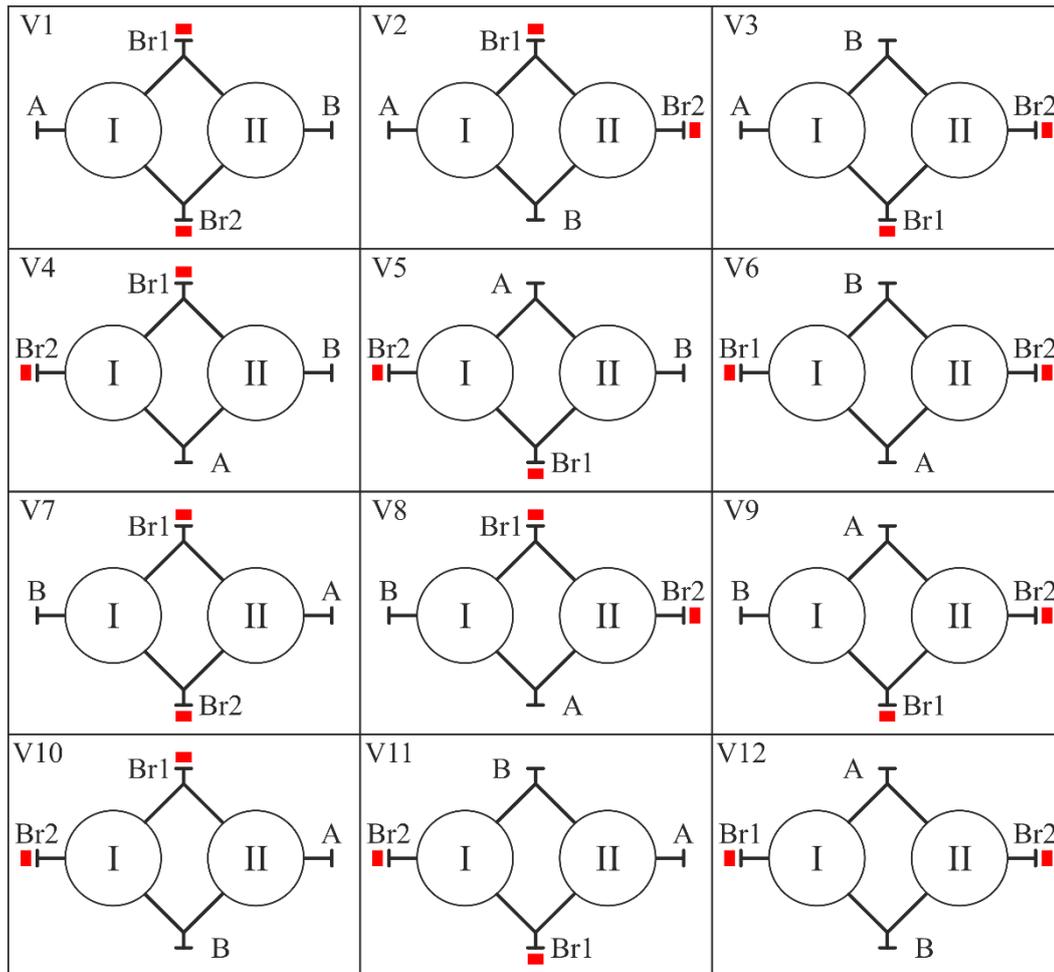


Figure 4. Systematization of layout variants. A – input shaft; B – output shaft; Br1, Br2 – brakes; V1-V12 – layout variants

2.2. The Compound Trains Process Examination

By placing the brakes on two shafts, a braking system is obtained in which the alternating activation of the brakes shifts the direction of the power flow through the planetary gear train, ultimately resulting in a variation of the transmission ratio.

Some compound planetary gear trains are depicted in (Jelaska, 2012; Troha, 2011; Troha et al., 2012, 2014; Arnaudov & Karaivanov, 2005), while the possible power flows are analysed, and transmission ratio at both component trains are derived in (Arnaudov & Karaivanov, 2001). The achievable range of transmission ratios and efficiencies of both component gear units is laid out in (Kudriavtsev & Kirdyashev, 1977), with 15 kinematic schemes being presented. A computer program for the choice of an optimal form of multi-speed PGTs is derived in (Troha et al., 2012), while the chart of changing capabilities for the realizable two-speed planetary gear trains was presented in (Troha, 2011).

There are three different groups of compound two-speed planetary gear trains in relation to the layout of the brakes. The first group has brakes placed on coupled shafts, the second group includes gear trains with brakes on single shafts while the third group of gear trains uses brakes on both coupled and single shafts. The actual characteristics of all groups are presented in (Troha, 2011).

The power flow through the compound gear trains with brakes on single shafts (V6 and V12) is laid out in Fig. 5.

All realizable variants of PGT with brakes on single shafts (layout variants V6 and V12) are symbolically laid out in Table 1. The left brake is activated to transfer power through the left component train (component train I), while the activation of the right brake, causes power to be transferred through the right component train (component train II).

The power input and output are through the coupled shafts. In this example, regardless of brake activation, the power is actively transmitted by only one component train, while the other remains idle. Hence, the

transmission ratios of the compound train are equal to the component gear trains transmission ratios. The transmission ratio ranges for both operating regimes are provided for all variants in Table 1.

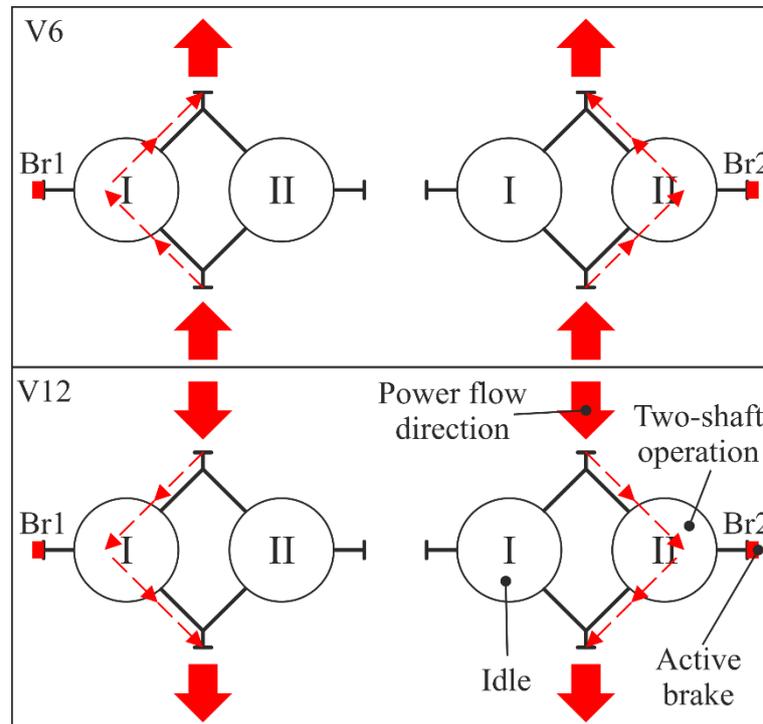


Figure 5. Power flows through the compound gear train with brakes on the single shafts

The transmission ratio range of every variant in each group is determined by its specific properties. Some variants will provide reduction or multiplication in both ratios, while others will offer reduction in one ratio, and multiplication in the other ratio. Likewise, some PGTs will have different directions of revolution of the output shaft, while others will keep the same direction.

The transmission ratios that can be achieved by these variants are limited only by the kinematic capabilities of their respective planetary gearsets, while the transmission ratio of each unit refers only to the ideal torque ratio of the active planetary gear unit. Therefore, such PGTs can provide an adequate solution if the required transmission ratios i_1 and i_2 can be achieved with a single gear unit.

All the possible kinematic schemes have been determined by means of the computer program DVOBRZ. As this paper deals with transmissions in which brakes are placed on single shafts, those transmissions are singled out and presented in Table 1. In addition to kinematic schemes, the transmission ratios with either brake 1 or brake 2 activated are shown. Note that brake 1 (Br1) is mounted on gearset I, while brake 2 (Br2) is mounted on gearset II. S denotes schema and V layout variant.

Some shapes have rather inviting features, significant for determining their possible field of application. For example, layout S36V6 shifts the direction of the output member revolution by shifting the transmission ratio. Therefore, this PGT is suitable for a machine tool, which has a high load, low speed working motion, and a fast, low resistance return motion to increase productivity. This layout also gives equal and opposite output shaft speeds with $t_1 = 1 + t_{II}$. Layouts S34 and S56 may be used for a transmission with inverse ratios, the ideal torque ratios $t_1 = t_{II}$ being equal.

It must be mentioned that PGTs where brakes are situated on single shafts have some design limitations. For example, a layout using three planets per units cannot achieve transmission ratios lower than 0,0769 or greater than 13, and PGTs where brakes are situated on coupled shafts or with brakes on coupled and single shafts should be considered for such cases.

Table 1. Design layouts and kinematic features of PGTs with brakes on single shafts

<p>S11</p>		<p>S12</p>	
<p>S11V6 Power flow: $X \rightarrow Y$ $i_{Br1} \in (1,083...1,5)$ $i_{Br2} \in (1,083...1,5)$</p>	<p>S11V12 Power flow: $Y \rightarrow X$ $i_{Br1} \in (0,923...0,666)$ $i_{Br2} \in (0,923...0,666)$</p>	<p>S12V12 Power flow: $X \rightarrow Y$ $i_{Br1} \in (0,923...0,666)$ $i_{Br2} \in (1,083...1,5)$</p>	<p>S12V6 Power flow: $Y \rightarrow X$ $i_{Br1} \in (1,083...1,5)$ $i_{Br2} \in (0,923...0,666)$</p>
<p>S13</p>		<p>S14</p>	
<p>S13V6 Power flow: $X \rightarrow Y$ $i_{Br1} \in (1,083...1,5)$ $i_{Br2} \in (3...13)$</p>	<p>S13V12 Power flow: $Y \rightarrow X$ $i_{Br1} \in (0,923...0,666)$ $i_{Br2} \in (0,333...0,077)$</p>	<p>S14V12 Power flow: $X \rightarrow Y$ $i_{Br1} \in (0,923...0,666)$ $i_{Br2} \in (3...13)$</p>	<p>S14V6 Power flow: $Y \rightarrow X$ $i_{Br1} \in (1,083...1,5)$ $i_{Br2} \in (0,333...0,077)$</p>
<p>S15</p>		<p>S16</p>	
<p>15V12 Power flow: $X \rightarrow Y$ $i_{Br1} \in (0,923...0,666)$ $i_{Br2} \in (-2...-12)$</p>	<p>15V6 Power flow: $Y \rightarrow X$ $i_{Br1} \in (1,083...1,5)$ $i_{Br2} \in (-0,5...-0,083)$</p>	<p>16V6 Power flow: $X \rightarrow Y$ $i_{Br1} \in (1,083...1,5)$ $i_{Br2} \in (-2...-12)$</p>	<p>16V12 Power flow: $Y \rightarrow X$ $i_{Br1} \in (0,923...0,666)$ $i_{Br2} \in (-0,5...-0,083)$</p>

Table 1. Design layouts and kinematic features of PGTs with brakes on single shafts (continued)

<p>S33V6 Power flow: $X \rightarrow Y$ $i_{Br1} \in (3...13)$ $i_{Br2} \in (3...13)$</p>	<p>S33V12 Power flow: $Y \rightarrow X$ $i_{Br1} \in (0,333...0,077)$ $i_{Br2} \in (0,333...0,077)$</p>	<p>S34V6 Power flow: $X \rightarrow Y$ $i_{Br1} \in (3...13)$ $i_{Br2} \in (0,333...0,077)$</p>	<p>S34V12 Power flow: $Y \rightarrow X$ $i_{Br1} \in (0,333...0,077)$ $i_{Br2} \in (3...13)$</p>
<p>S35V6 Power flow: $X \rightarrow Y$ $i_{Br1} \in (3...13)$ $i_{Br2} \in (-0,5...-0,083)$</p>	<p>S35V12 Power flow: $Y \rightarrow X$ $i_{Br1} \in (0,333...0,077)$ $i_{Br2} \in (-2...-12)$</p>	<p>S36V6 Power flow: $X \rightarrow Y$ $i_{Br1} \in (3...13)$ $i_{Br2} \in (-2...-12)$</p>	<p>S36V12 Power flow: $Y \rightarrow X$ $i_{Br1} \in (0,333...0,077)$ $i_{Br2} \in (-0,5...-0,083)$</p>
<p>S55V12 Power flow: $X \rightarrow Y$ $i_{Br1} \in (-2...-12)$ $i_{Br2} \in (-2...-12)$</p>	<p>S55V6 Power flow: $Y \rightarrow X$ $i_{Br1} \in (-0,5...-0,083)$ $i_{Br2} \in (-0,5...-0,083)$</p>	<p>S56V12 Power flow: $X \rightarrow Y$ $i_{Br1} \in (-2...-12)$ $i_{Br2} \in (-0,5...-0,083)$</p>	<p>S56V6 Power flow: $Y \rightarrow X$ $i_{Br1} \in (-0,5...-0,083)$ $i_{Br2} \in (-2...-12)$</p>

3. Mathematical Model for Planetary Gear Train Optimization

Multi-criteria optimization was referred to the compound planetary gear trains which enable two-speeds by brakes placed on single shafts discussed in this paper. These compound transmissions (Fig.1) are built up from planetary gear trains of the basic type as shown in Fig 2.

The planetary unit shown in Fig. 2 is a design with an externally geared sun gear 1, an internally geared ring gear (annulus) 3, externally geared planet gears 2, and planet carrier h. The planets are in synchronic touch with the sun and ring gear. The multi-criteria optimization is restricted to geared pairs in mesh.

The process of optimization prefaces with the explication of the mathematical model (Stefanović-Marinović et al., 2011). The total mathematical model of the basic type of a PGT is depicted in the previously mentioned paper and will be shortly covered in this chapter. It is necessary to determine the variables, objective functions, and functional constraints for mathematical model depiction.

3.1. Variables

Since each objective function is the function of several parameters, it is required to determine the variables.

The following variables are taken into consideration by the mathematical model: the number of teeth of the sun gear z_1 , the number of teeth of the planets z_2 , the number of teeth of the ring gear z_3 , the number of planets n_w , the gear module m_n , and the gear face width b .

The variables are of the hybrid type, discrete and continual: the numbers of gear teeth (z_1, z_2, z_3) are integers, the number of planets (n_w) is a discrete value, the module (m_n) is a discrete standard value (acc. to ISO 54 (DIN 780)), while the face width (b) is a continual variable. Also, some of them are dimensional (the face width and module are shown in millimetres) and others are non-dimensional (the numbers of gear teeth and the number of planets).

3.2. Objective Functions

The following characteristics have been chosen for the objective functions: volume, mass, efficiency, and manufacturing cost of gear pairs (Stefanović-Marinović, 2008).

The overall dimension represents the volume of the gear pairs. For that purpose, the gear is approximated by a cylinder with the diameter equal to the pitch diameter and the height equal to the face width. As the planets are inside the ring gear, this allows the gear volume to be stated by Eq. (4) (Stefanović-Marinović, 2008):

$$V = \frac{\pi}{4} \cdot b \cdot \left(\frac{m_n \cdot z_3}{\cos \beta} \cdot \frac{\cos \alpha_t}{\cos \alpha_{wt23}} \right)^2 \quad (4)$$

where α_t is the transverse pressure angle, α_{wt23} is the working transverse pressure angle for the pair 2-3 and β is the helix angle at the pitch diameter.

The *mass* represents the sum of all gear masses in a gear train. Since the mass of each gear is determined as gear volume magnified by the density of gear material, this criterion takes the last expression, given by Eq. (5):

$$m = 0.25 \cdot \pi \cdot b \cdot \rho \cdot \frac{m_n^2}{\cos^2 \beta} \cdot \left[k_1 \cdot z_1^2 \cdot \frac{\cos^2 \alpha_t}{\cos^2 \alpha_{wt12}} + \right. \\ \left. + n_w \cdot k_2 \cdot z_2^2 \cdot \frac{\cos^2 \alpha_t}{\cos^2 \alpha_{wt12}} + k_3 \cdot z_3^2 \cdot \frac{\cos^2 \alpha_t}{\cos^2 \alpha_{wt23}} \right] \quad (5)$$

One very significant design criteria and crucial factor in the evaluation of the design quality is the *efficiency*. Power losses in planetary gearsets can be classified as gear flank contact losses, bearing losses and oil viscosity losses. The computation of the gear trains efficiency is commonly limited to losses related to tooth flank friction, i.e., on the determination of contact power losses (Stefanović-Marinović, 2008; Stefanović-Marinović et al., 2011). Then, the expression for efficiency receives form:

$$\eta = \frac{1 - i_0 \cdot \eta_0}{1 - \eta_0} \quad (6)$$

where η_0 represents the efficiency with the immovable planet carrier determined by Eq. (7):

$$\eta_0 = 1 - \frac{z_3}{z_3 - z_1} \left[\frac{0.15}{z_1} + \frac{0.35}{z_2} + \frac{0.20}{z_3} \right] \quad (7)$$

and i_0 represents the basic transmission ratio $i_0 = z_3 / z_1$.

The techno-economical optimization needs to be considered also economic demands.

Initially, these requirements are affected by *manufacturing costs*. They comprise the rawmaterial and the manufacturing process costs. As a quantity of the production costs and as an economic factor, the time for gear manufacturing is accepted. The function which defines manufacturing costs as a sum of time periods required to produce the sun gear (T_{P1}), the planet gears (T_{P2}) and the ring gear (T_{P3}) is:

$$F_T = T_{P1} + n_w T_{P2} + T_{P3} \quad (8)$$

The production times have been defined with reference to the techniques of Fette, Lorenc and Höfler (Stefanović-Marinović, 2008).

3.3. Functional Constraints

Planetary gear trains must fulfil additional constraints to operate correctly, notably the constraints related to assembly, geometry, and strength.

The assembly constraints of coaxiality, adjacency and conjunction (Niemann and Winter, 1989), are well known and will not be discussed in detail.

The geometrical constraints refer to the calculation of characteristic geometrical values and comparison to the limitations of value ranges. The respect of these constraints is provided in concordance with the actual standards (ISO TC 60 list of standards 090915).

The strength constraints, notably by means of the safety factors for bending strength and surface durability of each gear, have been taken into consideration according to ISO 6336:2006 (ISO, 2006).

3.4. Optimization Process

The postulate of the optimization process shown in this study is the correlation of solutions with distinct parameters under identical conditions and the choice of the optimal variant. The optimization process starts with the generation of all results for the given input data. For the selected input data, all combinations of 6 design parameters ($z_1, z_2, z_3, n_w, m_n, b$) fulfilling the functional constraints are founded and objective function values for every 6-tuple are calculated. These 6-tuples create a set of feasible solutions.

The optimal solution defined by design parameters is chosen related to accepted objective functions, and the multicriteria nonlinear task can be expressed by the following equation:

$$\max \{f_1(x), f_2(x), \dots, f_k(x)\} \forall x \in S \quad (9)$$

In which which $f_1(x), \dots, f_k(x)$ are *objective functions*, $x = (x_1, \dots, x_n)$ is *decision variables vector* and S is the *feasible solutions* set. Every point $x \in S$ is joined to a point $(f_1(x), f_2(x), \dots, f_k(x))$ in k - dimensional objective space. Hence, the objective set can be defined:

$$F = \{(f_1(x), f_2(x), \dots, f_k(x)) \mid x \in S\} \quad (10)$$

The notation "max" relates to synchronous maximization of volume, mass, efficiency, and manufacturing costs. It should be noted that, minimization of the function $f_i(x)$ is identical to the maximization of the function $-f_i(x)$. Having in mind the structure of feasible solution set S , *discrete* multi-criteria optimization tasks exist. There are six variables in this optimisation task, correlated to the adopted design parameters: $x = (x_1, x_2, x_3, x_4, x_5, x_6) = (z_1, z_2, z_3, n_w, m_n, b)$. Additionally, the objective functions are volume $V(x)$, mass $m(x)$, efficiency $\eta(x)$ and manufacturing costs $T(x)$:

$$f_1(x) = -V(x), f_2(x) = -m(x), f_3(x) = \eta(x), f_4(x) = -T(x) \quad (11)$$

Then, the formulation of the multicriteria mathematical model for the actual optimization task can be expressed as:

$$\max \{f_1(x), f_2(x), f_3(x), f_4(x)\} \forall x \in S \quad (12)$$

It can be seen from the previous definition that multi-criteria optimization problems are unwell determinate. The *Pareto optimality concept* is the well-known concept for making choices between "equally good" solutions. The solution $x \in S$ is Pareto optimal if there is no solution $y \in S$ such that holds $f_i(x) \leq f_i(y)$ for all $i = 1, \dots, n$, and for at least one index i holds strict inequality, i.e., $f_i(x) < f_i(y)$. The first step is the determination of the Pareto optimal solutions set, while the next stage in the process of optimal solution determination is the selection of the optimal solution choice from the Pareto set. The weighted coefficients method is used for that purpose.

3.4.1. *Weighted Coefficients Method*

This method belongs to the group of methods which scalarize multicriteria problems. By using this method, the scalarized problem is:

$$\max f^M(x) = w_1 \cdot f_1^0(x) + \dots + w_m \cdot f_m^0(x) \forall x \in S \tag{13}$$

where w_i are weighted coefficients and $f_i^0(x) = (f_i^*)^{-1} \cdot f_i(x)$ whereby f_i^0 are normalizing coefficients.

The ideal point $f^* = (f_1^*, f_2^*, f_3^*, f_4^*)$ components are applied like normalizing coefficients in this model, i.e., $f_i^0 = f_i^*$ for $i=1,2,3,4$. Hence, absolute values are between 0 and 1 for all objective functions, which simplifies the selection of the weighted coefficients. Solutions realized by weighted coefficients method are Pareto optimal (Stefanović-Marinović et al., 2011). Extensive experience of the weighted coefficients method is available, specially in technical systems optimization. This model is suitable for cases with priority or equal priority functions (Stefanović-Marinović, 2008).

A simplified flowchart of the complete optimisation procedure is shown in Fig. 6. The whole optimization process is a part of *PlanGears* software.

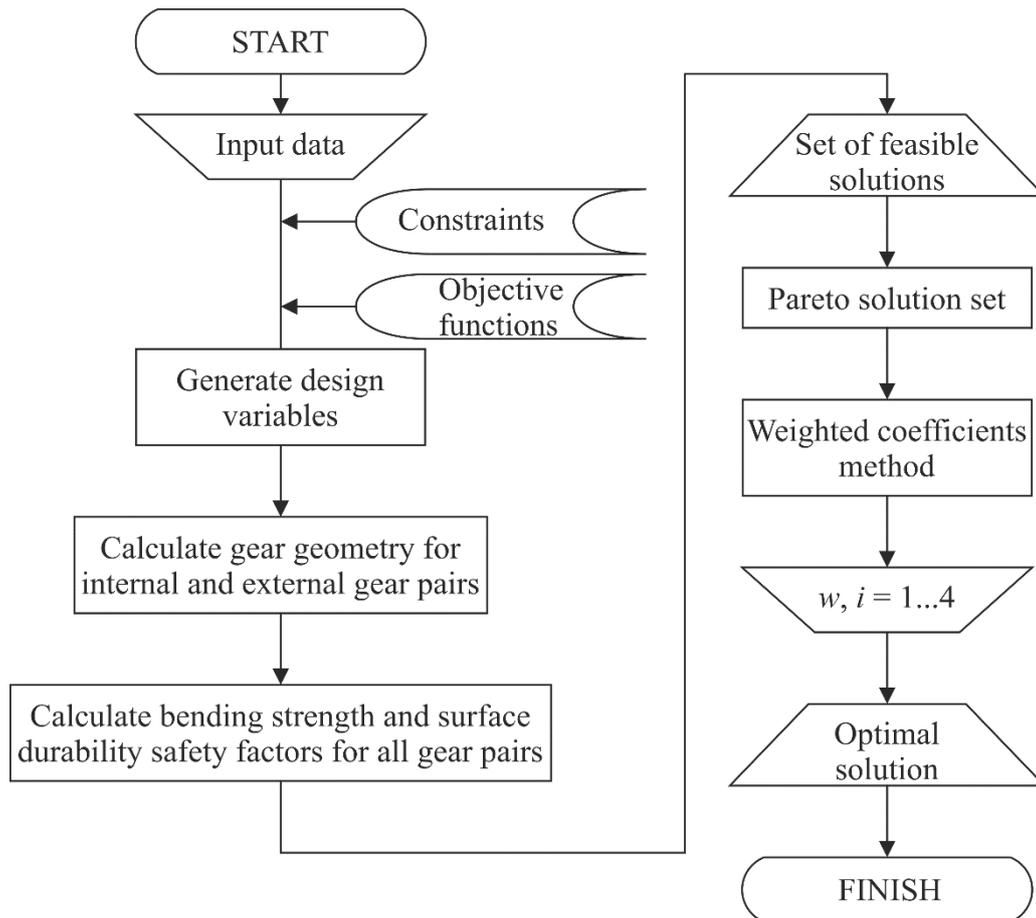


Figure 6. Simplified flowchart of the optimization procedure

4. Results and Discussion

Multicriteria optimization can be applied to every PGT type mentioned in Table 1 while keeping their specifics in consideration.

First, the working demands involved require transmission ratios $i = 6$ in one direction ($i_{Br1} = 6,0$) and $i = 0,3$ ($i_{Br2} = 0,3$) in the other direction, and therefore only one variant from Table 1 can be chosen. It is variant S34V6, presented symbolically and kinematically in Fig. 7.

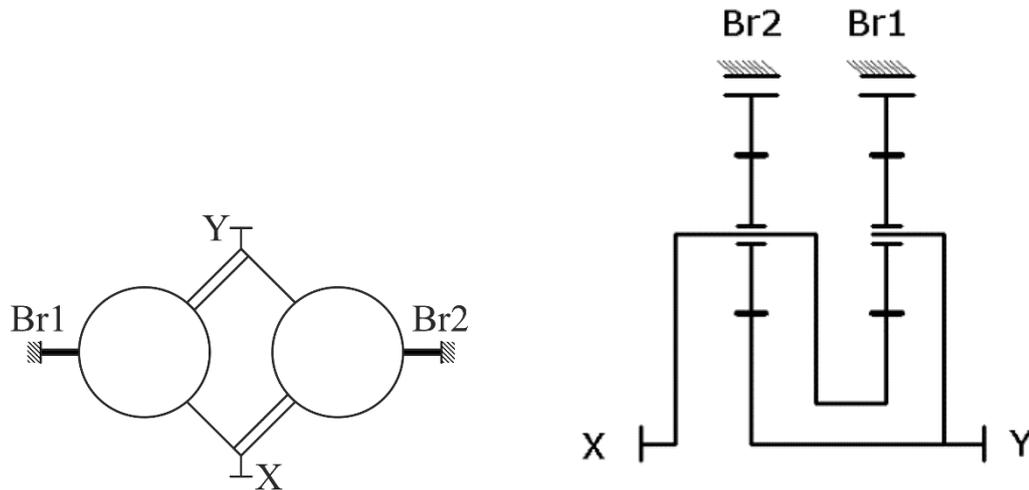


Figure 7. Symbolic presentation of PGT (left), kinematic scheme (right)

The direction of the power flow is presented in Fig. 8. This type of transmission has applications in mixers for the chemical industry. The input data for optimal solution selection is adopted according to working conditions.

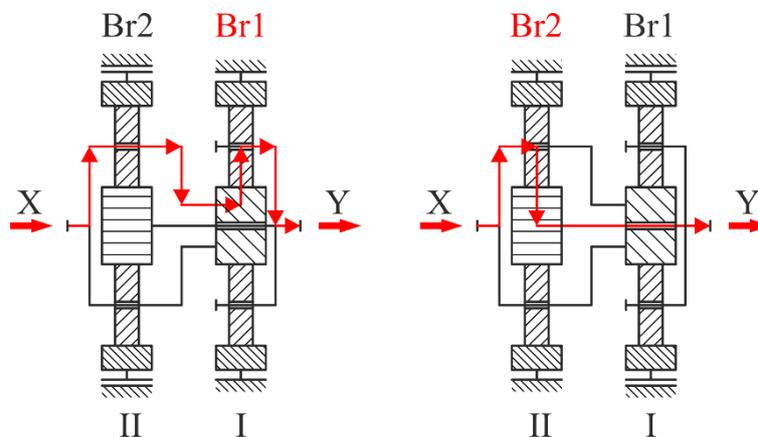


Figure 8. Power flow through the PGT with active brake Br1 (left) and brake Br2 (right)

4.1. First Component Gear Train (I)

With only brake Br1 activated, the ring gear of the first component train is immovable. The input is through the carrier of the second component gear train and sun gear of the first component gear train. Because only the stationary element is reactive, the second component train idles as it has no resistance. The first component train is defined in this order.

The important required input data is: $i = 6$, $n_{in} = 1500 \text{ min}^{-1}$, $P = 20 \text{ kW}$, $L = 1000 \text{ h}$, $K_A = 1,1$, IT7 for all gears, sun and planet gear material is 20MoCr4, while 34CrNiMo6 was predicted for ring gear.

The feasible set contains 788 solutions, but only 37 solutions are Pareto solutions. Since it is determined that the first criterion (mass) and the fourth criterion (manufacturing costs) have significant influence, the optimal solution is adopted related to the first and the fourth criteria and the following weighted coefficients

are adopted: $w_1 = w_4 = 0,5$, $w_2 = w_3 = 0,0$. The solution given in Table 2 was obtained by means of the weighted coefficient method, while the objective function values of this solution are given in Table 3.

Table 2. Optimal solution related to the first and fourth criteria

Variable values					
$x_1 = z_1$	$x_2 = z_2$	$x_3 = z_3$	$x_4 = n_w$	$x_5 = m_n$	$x_6 = b$
15	28	-73	4	2,25	18

Table 3. Objective function for solution from Table 2

f_1 [mm ³]	f_2 [kg]	f_3	f_4 [min]
372034,38	2,02	0,983	108,74

4.2. Second Component Gear Train (II)

With only brake Br2 activated, the input data is: $i = 0,3$, $i' = \frac{1}{i} = \frac{1}{0,3} = 3,33$, $n_{in} = 1500 \text{ min}^{-1}$, $T_{in} = 131,7 \text{ Nm}$ ($P = 20 \text{ kW}$), $L = 1000 \text{ h}$, $K_A = 1,1$, IT7 for all gears, material for sun and planet gear is 20MoCr4, while it is predicted that ring gear will use 34CrNiMo6.

The feasible set consists of 7 solutions, while the number of Pareto solutions is 3. The solution given in Table 4 with objective function values given in Table 5 was obtained by means of the weighted coefficient method with coefficient values: $w_1 = 0,5$, $w_2 = 0,0$, $w_3 = 0,0$, $w_4 = 0,5$.

Therefore, it is obvious that Pareto optimality access for selecting between equally valid solutions can be sensibly applied to compound PGT in relation to the adopted optimization criteria.

The same procedure can be applied to other types of compound PGTs. Other, additional criteria must be used in the process of selecting optimal solutions for other applications.

Table 4. Optimal solution obtained by weighted coefficient method

Variable values					
$x_1 = z_1$	$x_2 = z_2$	$x_3 = z_3$	$x_4 = n_w$	$x_5 = m_n$	$x_6 = b$
32	21	-73	3	2	15
Variable values					
$x_1 = z_1$	$x_2 = z_2$	$x_3 = z_3$	$x_4 = n_w$	$x_5 = m_n$	$x_6 = b$
15	28	-73	4	2,25	18

Table 5. Objective function for solution from Table 4

f_1 [mm ³]	f_2 [kg]	f_3	f_4 [min]
260874,66	1,427	0,987	78,12

4. Conclusion

This paper introduces the process of fast resolution of the internal structure and design parameters of two-speed compound gear trains. This is achieved by using two computer programs: *DVOBRZ* expanded for examination of the compound gear trains and *PlanGears*, developed for the application of multicriteria optimization to planetary gear trains.

The acceptable solution is given complete with kinematic schemes and symbolic view with power flow. Also, there are design parameters for acceptable solutions obtained by considering mass and manufacturing costs, which are applied by means of the Pareto optimization with weighted coefficient method.

This concept is a novelty in planetary gear train optimization and can be beneficial when applied to both the component and compound gear trains, as presented in this paper. The conclusions resulting from the use of the procedure shown in this paper are in conformity with available sources on mechanical system optimization and confirm the appropriate selection of the applied methods.

In addition, this also proves that this method can be applied in the selection of optimal solutions for other planetary gear trains.

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