Finite element thermal analysis of a moving porous fin with temperature-variant thermal conductivity and internal heat generation

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ABSTRACT

This paper focuses on finite element analysis of the thermal behaviour of a moving porous fin with temperature-variant thermal conductivity and internal heat generation. The numerical solutions are used to investigate the effects of Peclet number, Hartmann number, porous and convective parameters on the temperature distribution, heat transfer and efficiency of the moving fin. The results show that when the convective and porous parameters increase, the adimensional fin temperature decreases. However, the value of the fin temperature is amplified as the value Peclet number is enlarged. Also, an increase in the thermal conductivity and the internal heat generation cause the fin temperature to fall and the rate of heat transfer from the fin to decrease. Therefore, the operational parameters of the fin must be carefully selected to avoid thermal instability in the fin.

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1. Introduction

Heat transfer enhancements in thermal systems, mechanical processes, electronic gadgets, chemical processes, etc. have been passively and effectively achieved by the application of extended surfaces, called fins and spines. The importance of such fins in these various systems aroused various studies. In these studies, Kiwan and Al-Nimir (2000) pointed out the use of fins with pores for thermal augumentation of the extended surfaces. Gong et al (2018) illustrated the use of porous and solid compound fins for heat sink in micro-channel. Ali et al. (2018) experimentally investigated the effects of heat sink fin shapes and phase change materials on the thermal cooling of electronics. Saedodin (2011) presented numerical analysis of temperature distribution in natural convection fins with pores while Sobamowo et al. (2018) applied Galerkin method of weighted residual for the analysis of the temperature distribution in the natural convection porous fins. Oguntala et al. (2017) explored the impact of particle deposition on the thermal bahaviour of heat sink porous fins with convective and radiative heat transfer. Mosayebidorcheh et al. (2014) investigated the transient response of fins with different shapes and variable thermal properties and internal heat generation. Kim and Mudawar (2010) also examined the shape effects on heat diffusion through a heat sink. Moradi et al. (2014) adopted differential transformation method to analyze fins with triangular profiles and temperature-variant thermal conductivity. The method provided solution in the form of infinite power series with high accuracy.

Various key parameters in the porous fin thermal models have been used for the improved heat transfer enhancement (Kiwan et al., 2000; Gong et al., 2018; Ali et al., 2018; Seyfolah, 2011; Oguntala et al., 2018). Some of the past works have focused on the utilizations of the fin geometry as well as the thermo-electro-magnetic properties of fin to achieve the optimized heat transfer augmentation of the porous fin (Mosayebidorcheh et al., 2014; Kim and Mudawar, 2010; Moradi et al., 2014; Oguntala et al., 2018; Wan et al., 2012; Naphon et al., 2009; Oguntala et al., 2018; Sobamowo, 2016). In some of the studies, the properties of the surrounding fluid around the passive device have been used to increase the heat dissipating capacity of the fin (Seyf and Feizbakhshi, 2012; Fazeli et al., 2012; Oguntala et al., 2017). Additionally, some authors displayed the efficiency of some new analytical and numerical methods in the thermal analysis of the porous fin (Kundu and Bhanja, 2011; Khani et al., 2009; Rostamiyan et al., 2014; Das and Kundu, 2017; Oguntala et al., 2018; Oguntala and Abd-Alhameed, 2017). Thermal behaviours of porous extended surface are studied in other works (Kiwan, 2007a, 2007b, 2008; Gorla and Bakier, 2011; Kundu and Bhanji, 2011; Kundu et al., 2012; Taklifi et al., 2010; Oguntala et al., 2019, 2020).

The study of thermal behavior of continuous moving surfaces such as extrusion, hot rolling, glass sheet or wire drawing, casting, powder metallurgy techniques for the fabrication of rod and sheet has become an area of increasing research interests. In the processes such as rolling of strip, hot rolling, glass fiber drawing, casting. extrusion, drawing of wires and sheets, there is an exchange of heat between material and the surroundings while the material moves through the roller or the furnace as shown in Figure 1.

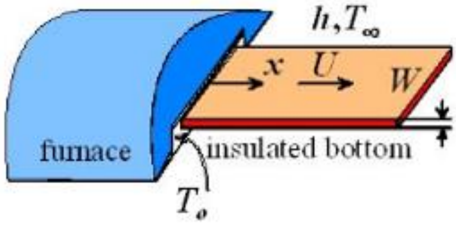

Figure 1. Schematic diagram of extrusion process

Since the operations and the thermal configuration of these metal fabrication and forming technologies satisfy the criterion for fin approximation, they can therefore be thermal modeled as continuously moving fins. Due to these adaptable and wide areas of applications, there have been extensive research works on the continuous moving fins. Moreover, in industrial processes, control of cooling rate of the sheets is very important to obtain desired material structure. Consequently, various studies on heat transfer and thermal analysis of a continuously moving fin have been presented in literature. In such analysis of the extended surfaces, Torabi et al. (2012) developed the analytical solutions for convective-radiative heat transfer in a continuously moving fin with variable thermal conductivity while Aziz et al. (2011) studied the convection–radiation heat transfer from a continuously moving surface with variable thermal conductivity. Aziz and Khani (2011) adopted homotopy analysis method to analyze the same problem. With the aid of wavelet collocation method, Singh et al. (2013) examined the thermal characteristics of a convective-radiative continuously moving fin with temperature-variant thermal conductivity. Also, Aziz and Torabi (2012) explored the thermal behaviour of a moving fin with temperature-variant thermal properties. The effectiveness of spectral element method was demonstrated in the study carried out by Ma et al. (2016) on thermal behaviour of longitudinal porous moving fins of different profiles. Sun et al. (2015) predicted the heat transfer in convective-radiative moving rod using spectral collocation method while Aziz and Khani (2011) adopted homotopy analysis method to analyze the same problem. Aziz and Lopez (2011) presented the numerical investigation of the convective-radiative moving fin. Torabi et al. (2012) utilized differential transformation method for the continuously moving fin losing heat through both convection and radiation and having temperature-dependent thermal conductivity. In another study, Kanth and Kumar (2013) applied Haar wavelet method for the moving convective-radiative Fin with variable thermal conductivity. Singla and Ranjan (2014) used Adomian decomposition method for the inverse heat transfer problems in a moving fin. Moradi and Rafiee (2003) developed analytical solution using differential transformation method for the convection-radiation heat transfer in a continuously moving fin with temperature-variant thermal conductivity. Dogonchi and Ganji (2016) considered the flow of convection-radiation heat transfer in moving fin with variable thermal properties and internal heat generation while Sun and Ma (2015) used collocation spectral method to theoretically investigate the same problem. Singh et al. (2013) analyzed convective-radiative moving fin with temperature-variant thermal conductivity using wavelet collocation approach. With the application of simplex search method, Ranjan (2011) explored the thermal behaviour of a conductive-convective fin with variable conductivity.

To the best of the authors' knowledge, the thermal analysis of heat transfer in a moving convective porous fin with temperature-dependent thermal conductivity and internal heat generation using finite element method has not been carried out. Therefore, the thermal analysis of a moving porous fin with temperature-variant thermal conductivity and internal heat generation is studied using finite element method is carried out. Parametric studies are presented using the numerical solutions. The numerical solutions are used to investigate the effects of Peclet number, Hartmann number, convective and porous parameters on the thermal performance of the moving fin.

2. Development of the Thermal Model

Consider a rectangular moving fin in Figure 2. The moving fin is convecting and radiating heat with temperature-variant thermal conductivity and internal heat generation. The extended surface is exposed to an environment with thermal properties as shown in the Figure. It is assumed that the temperature varies in the fin is only along the length of the fin. and there is a perfect contact between the fin base and the prime surface.

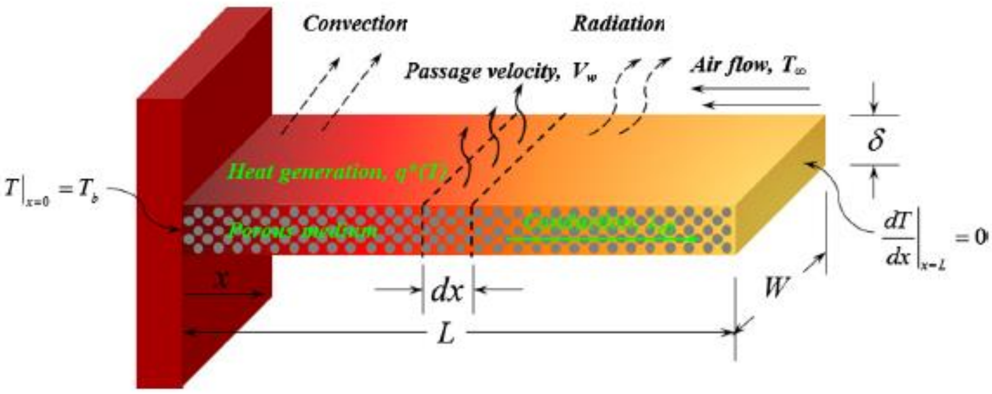

Figure 2. Schematic of the convective-radiative moving longitudinal straight fin (Ma et al., 2016)

Form the assumptions and the energy balance to an elemental section of the fin profile Δx and using Darcy formulation for the porous medium,

$$\begin{pmatrix} rate\, of\ heat \\ Conducted\ \text{into} \\ \text{the element}\ \Delta x \end{pmatrix} - \begin{pmatrix} rate\, of\ heat \\ Conducted\ from \\ \text{the element}\ \Delta x \end{pmatrix} - \begin{pmatrix} rate\, of\ heat\, loss \\ due\, to\, Porosity \\ of\ the\, \text{element}\ \Delta x \end{pmatrix}$$
$$- \begin{pmatrix} rate\, of\ heat \\ Convection\ from \\ \text{the element}\ \Delta x \end{pmatrix} - \begin{pmatrix} heat\ Advection \\ due\, to\, motion\, of \\ \text{the element}\ \Delta x \end{pmatrix} + \begin{pmatrix} rate\, of\ \text{internal} \\ heat\ generation \\ \text{in the element}\ \Delta x \end{pmatrix} = 0 \tag{1}$$

$$q(x) - q(x+\Delta x) - \dot{m} c_p (T - T_\infty) - h(P.\Delta x)(T - T_\infty) - \rho c_p V_w A \Delta x \frac{dT}{dx} + q^* A \Delta x = 0 \tag{2}$$

As $\Delta x \to 0$, equation (3) becomes

$$-\frac{dq}{dx} - \frac{\dot{m} c_p (T - T_\infty)}{\Delta x} - hP(T - T_\infty) - \rho c_p V_w A \frac{dT}{dx} + q^*(A) = 0 \tag{3}$$

The fluid passes through the porous material at a mass flow rate given by

$$\dot{m} = \rho V_w \Delta x.w \tag{4}$$

From Darcy's model, we have that

$$V_w = \frac{gk\beta^*\left(T - T_\infty\right)}{\nu}\ . \tag{5}$$

Substituting Eq. (5) into Eq. (3), gives

$$-\frac{dq}{dx} - \frac{\rho c_p gk\beta^* w\left(T - T_\infty\right)^2}{\nu} - hP\left(T - T_\infty\right) - \rho c_p V_w A\frac{dT}{dx} + q_o^*\left(T\right)A = 0 \tag{6}$$

From Fourier law heat of conduction,

$$\frac{dq}{dx} = \frac{d}{dx}\left(-k\left(T\right)A\frac{dT}{dx}\right) \tag{7}$$

where,

$$k\left(T\right) = k_0\left(1 + \lambda\left(T - T_\infty\right)\right) \tag{8}$$

and

$$q^*\left(T\right) = q_o^*\left(1 + \varepsilon\left(T - T_\infty\right)\right) \tag{9}$$

Substituting Eqs. (8), (9) and (10) into Eq. (7), gives

$$\frac{d}{dx}\left(\left(1 + \lambda\left(T - T_\infty\right)\right)\frac{dT}{dx}\right) - \frac{\rho c_p gk\beta^*\left(T - T_\infty\right)^2}{\nu k_o \delta} - \frac{hP\left(T - T_\infty\right)}{k_o A} - \frac{\rho c_p V_w}{k_o A}\frac{dT}{dx} + \frac{q_o^*\left(1 + \varepsilon\left(T - T_\infty\right)\right)}{k_o A} = 0 \tag{10}$$

Subject to the following boundary conditions;

$$T\big|_{x=0} = T_b \tag{11}$$

$$\left.\frac{dT}{dx}\right|_{x=L} = 0 \tag{12}$$

Introducing the following dimensionless variables,

$$\begin{gathered} X = \frac{x}{L},\quad \theta = \frac{T - T_\infty}{T_b - T_\infty},\qquad M^2 = \frac{hpL^2}{k_o A}\quad Sp = \frac{gk\beta^* L^2 (T_b - T_\infty)}{\nu\alpha\delta} = \frac{Da\, x\, Ra}{K}\left(\frac{L}{t}\right)^2 \\ \alpha = \frac{k_o}{\rho c_p}\quad Pe = \frac{V_w L}{\alpha}\qquad \beta = \lambda(T_b - T_\infty)\quad H_t = \varepsilon(T_b - T_\infty),\quad G = \frac{q_o^*}{hP(T_b - T_\infty)} \end{gathered} \tag{13}$$

Substituting the dimensionless variables into equation (10), gives the non-dimensional governing equation

$$\frac{d^2\theta}{dX^2} + \beta\theta\frac{d^2\theta}{dX^2} + \beta\left(\frac{d\theta}{dX}\right)^2 - Sp\theta^2 - M^2\theta - Pe\frac{d\theta}{dX} + M^2 G(1 + H_t\theta) = 0 \tag{14}$$

The boundary conditions appear in dimensionless form as

$$\theta(0) = 1 \tag{15a}$$

$$\frac{d\theta(1)}{dX} = 0 \tag{15b}$$

3. Finite Element Method to the Thermal Model

The strong nonlinearity in Eq. (14) renders the development of exact analytical solution to the nonlinear equation very difficult. Therefore, Galerkin finite element method is used in this work to solve the nonlinear equation. The procedures of the finite element analysis are summarized as follows:

- discretization of the domain into elements;
- derivation of element equations;
- assembly of element equations;
- imposition of boundary conditions; and
- solution of assembled equations.

Following the procedures of finite element method, the entire computational domain of each profile is partitioned into "*r*" number of linear elements of equivalent size. A typical element is isolated as shown in Fig. 3.

Then variational formulation of the given problem over the typical element is constructed as given by Eq. (13).

$$\int_{X_i}^{X_j} w_v\left\{\frac{d^2\theta}{dX^2}+\beta\theta\frac{d^2\theta}{dX^2}+\beta\left(\frac{d\theta}{dX}\right)^2-S_p\theta^2-M^2\theta-Pe\frac{d\theta}{dX}+M^2G(1+H_t\theta)\right\}dX=0 \quad (16)$$

where w_v is the weight function corresponding to $\theta(X)$.

Using the Galerkin Finite Element Method, the shape function is equal to the weight function, Therefore, Eq. (13) is written as

$$\int_{X_i}^{X_j} N_v\left\{\frac{d^2\theta}{dX^2}+\beta\theta\frac{d^2\theta}{dX^2}+\beta\left(\frac{d\theta}{dX}\right)^2-S_p\theta^2-M^2\theta-Pe\frac{d\theta}{dX}+M^2G(1+H_t\theta)\right\}dX=0 \quad (17)$$

where $v=i,j$

An approximate solution of the variational problem is assumed and the element equations are generated by substituting the assumed solution in the formulation as shown from Eqs. (18) - (26).

The element matrix, which is called stiffness matrix, is constructed by using the element interpolation functions as stated as follows:

The finite element approximate solution for a two-node linear element in Fig. 3 is given as

$$\theta(X)=N_i(X)\theta_i+N_j(X)\theta_j \quad (18)$$

After derivation from the two-node linear element, we have

$$\theta(X,\tau)=\left(\frac{X_j-X}{X_j-X_i}\right)\theta_i+\left(\frac{X-X_i}{X_j-X_i}\right)\theta_j, \quad (19)$$

where

$$N_i = \frac{X_j - X}{X_j - X_i}, \quad N_j = \frac{X - X_i}{X_j - X_i}, \tag{20}$$

N_i and N_j are called shape/interpolation/test/basis functions.

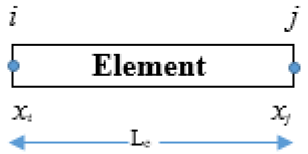

Figure 3. A 2-node element

For a two-node element as shown in Figure 3, with

$$X_i = 0, \quad X_j = L_e, \quad \Rightarrow X_j - X_i = L_e, \tag{21}$$

Therefore, the shape functions can be written as

$$N_i = 1 - \frac{X}{L_e}, \; N_j = \frac{X}{L_e}, \tag{22}$$

We can therefore write Eq. (19) as

$$\theta(X) = \left(1 - \frac{X}{L_e}\right)\theta_i + \left(\frac{X}{L_e}\right)\theta_j, \tag{23}$$

Then, the variational formulation can be written as

$$\int_0^{L_e} N_v \left\{ \frac{d^2\theta}{dX^2} + \beta\theta\frac{d^2\theta}{dX^2} + \beta\left(\frac{d\theta}{dX}\right)^2 - S_p\theta^2 - M^2\theta - Pe\frac{d\theta}{dX} + M^2G(1+H_t\theta) \right\} dX = 0 \tag{24}$$

which can be written as

$$\int_0^{L_e} N_v(X) \left\{ \frac{d^2\theta}{dX^2} + \beta\theta\frac{d^2\theta}{dX^2} + \beta\left(\frac{d\theta}{dX}\right)\left(\frac{d\theta}{dX}\right) - S_p\theta^2 - M^2\theta - Pe\frac{d\theta}{dX} + M^2G(1+H_t\theta) \right\} dX = 0 \tag{25}$$

On substituting Eq. (18) into Eq. (21), we have

$$\int_0^{L_e} \left\{ \begin{array}{l} -\left[\dfrac{dN_i(X)}{dX}\dfrac{dN_j(X)}{dX}\right] - \left[N_i(X)\dfrac{dN_j(X)}{dX}\dfrac{dN_j(X)}{dX} + \dfrac{dN_i(X)}{dX}\dfrac{dN_j(X)}{dX}\dfrac{dN_j(X)}{dX}\right] \\ -S_p\left(\bar{\theta}\right)^2 N_i(X)N_j(X) - M^2\left(\bar{\theta}\right)N_i(X)N_j(X) - PeN_i(X)\dfrac{dN_j(X)}{dX} + M^2GH_tN_i(X)N_j(X)\left(\bar{\theta}\right) \end{array} \right\} \theta dX \tag{26}$$

$$+\int_0^{L_e} M^2GN_i(X)dX + N_i(X)\theta\frac{dN_j(X)}{dX}\bigg|_0^{L_e} + \left(N_i\frac{d\theta}{dX}\right)\bigg|_0^{L_e} = 0$$

The above Eq. (26) can be written in the finite element model form as

$$\left[K^e\right]\left\{\theta^e\right\} = \left[f^e\right] \tag{27}$$

where

$$K_{ij}^e = -\int_0^{L_e} \left\{ \begin{array}{l} \left[\dfrac{dN_i(X)}{dX}\dfrac{dN_j(X)}{dX}\right] + \left[N_i(X)\dfrac{dN_j(X)}{dX}\dfrac{dN_j(X)}{dX} + \dfrac{dN_i(X)}{dX}\dfrac{dN_j(X)}{dX}\dfrac{dN_j(X)}{dX}\right] \\ +S_p\left(\bar{\theta}\right)^2 N_i(X)N_j(X) + M^2\left(\bar{\theta}\right)N_i(X)N_j(X) + PeN_i(X)\dfrac{dN_j(X)}{dX} - M^2GH_tN_i(X)N_j(X) \end{array} \right\} dX \tag{28}$$

$$f_i^e = \int_0^{L_e} M^2GN_i(X)dX + N_i(X)\theta\frac{dN_j(X)}{dX}\bigg|_0^{L_e} + \left(N_i\frac{d\theta}{dX}\right)\bigg|_0^{L_e} \tag{29}$$

where

$$\bar{\theta}(X) = N_i(X)\bar{\theta}_i + N_j(X)\bar{\theta}_j \tag{30}$$

5. Parameters of Engineering Interests in the Thermal analysis

The indication for the thermal performance analysis of fin is given by heat transfer and efficiency of the fin. Following the analysis in our previous study (Sobamowo, 2016),

The adimensional total heat flux of the fin is given by:

$$q_T = (1+\beta\theta)\frac{\partial\theta}{\partial X} \tag{31}$$

The fin efficiency

$$\eta = \frac{M^2\int_0^1 \theta dX + S_p\int_0^1 \theta^2 dX + Pe\int_0^1 \theta dX}{M^2 + S_p + Pe} \tag{32}$$

The non-dimensional total heat flux of the fin and the thermal efficiency can be found by substituting Eq. (19) into Eqs. (31) and (32) and then evaluate.

6. Results and Discussion

The developed analytical models are simulated in MATLAB and the results are given in Figures 4-15. Also, parametric and sensitivity analyses are carried out as presented and discussed. The impacts of porous, convective, moving, thermal conductivity and internal heat generation parameters on the adimensional temperature, heat transfer and efficiency of the moving fin are presented in Figures 4-15.

Figure 4 illustrates the impacts of porous parameter on the adimensional temperature in the moving porous fin. S_p is porous or porosity parameter which represents the product of Rayleigh number, Darcy number and aspect ratio. When S_p =0, it means that the fin is a solid fin. The figure shows that as porous parameter, S_p increases, the adimensional temperature in the fin decreases. These as the results of increase in the permeability and buoyancy force when the porosity or porous parameter increases. When this happens, it means that more pores are created that allow the working fluid to penetrate more through the pores of the fin. Consequently, the heat loss through the fin surface increases and the fin temperature falls, the rate of heat transfer from the fin and the efficiency of the fin are augmented as shown in Figure 15. However, it should be clearly stated that high value of porosity parameter decreases the effective thermal conductivity and ideal heat transfer.

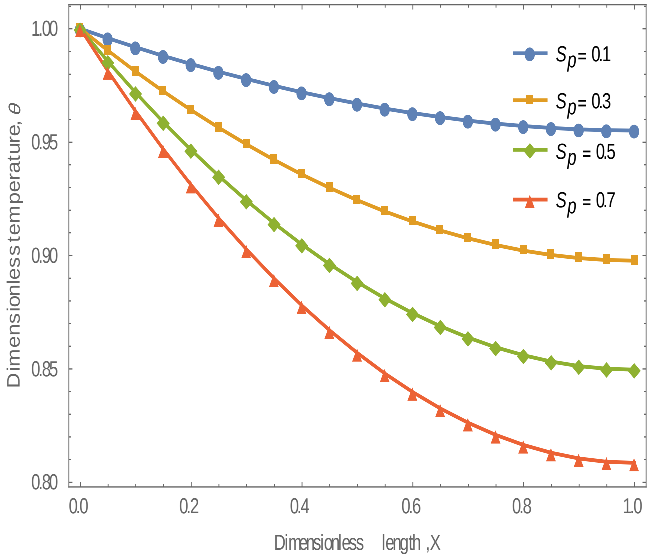

Figure 4. Effects of porosity parameter on the adimensional temperature when $\beta = 0.2; M = 0.3;$ Pe $= 0.5; G = 0.4; H_t = 0.6.$

Figure 5 present the influences of thermo-geometric or convective parameter on the adimensional temperature distribution in the fin, respectively. The figures show that as the conductive-convective parameter increases, the adimensional local temperature in the fin decreases. This is because, as the convective parameter increases, the effects of convective heat transfer on the fin surface increase, there is an increase in the heat loss from the surface of the fin surface. As a consequent, surface temperature of the fin drops (the fin thermal profile falls) and the rate of heat transfer from the fin increases as the convective and radiative parameters increase. It should be noted that the low value of the convective parameter, M implies a relatively thick and short fin of very high thermal conductivity while a high value of the convective parameter indicates a relatively thin and long fin of a very low thermal conductivity. Therefore, the rate of heat transfer from the fin and thermal efficiency of the fin are favoured at low values of convective parameter (as shown in Figure 10 and 14), i.e. a relatively thick and short fin with a high thermal conductivity.

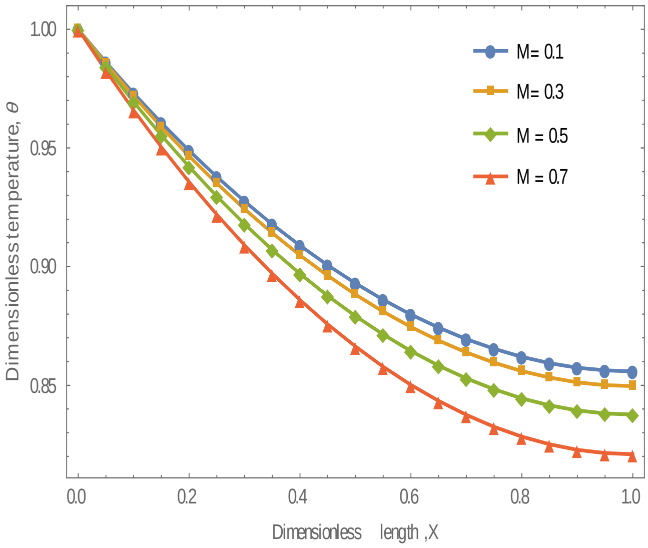

Figure 5. Effect of thermo-geometric parameter on the adimensional temperature when $\beta = 0.2$; $S_p = 0.5$; Pe $= 0.5$; $G = 0.4$; $H_t = 0.6$.

Influence of Peclet number on the adimensional temperature in th fin are shown in Figure 6. *Pe* represents the Peclet number which is the adimensional speed of the moving fin, When *Pe* =0, it means that the fin is a stationary fin. The values of the temperature distribution in the fin increases as the Peclet number increases. This is expected because with an increase in Peclet number, the material moves faster and the time for which the material is exposed to the environment gets shorter as well as the losing heat from fin surface gets stronger, thus the fin temperature increases.

Figure 7 presents the effects of thermal conductivity parameter, β on the adimensional temperature distribution in the fin. It is shown that the fin temperature is directly proportional to the fin thermal conductivity. Physically speaking, when the thermal conductivity parameter is amplified which results in an increase in the local temperature of the fin, the heat conduction process in the fin as well as the fin efficiency are enhanced as shown in Figures 10-13. It is further observed that the temperature of the fin-tip amplifies as the thermal conductivity parameter magnifies. Also, the best fin efficiency can be gotten when thermal conductivity parameter β is kept as low as possible.

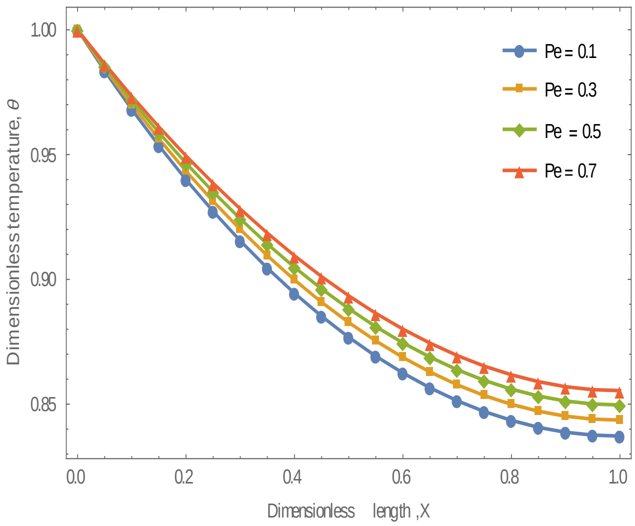

Figure 6. Effect of Peclet number on the adimensional temperature when $\beta = 0.2$; $M = 0.3$; $S_p = 0.5$; $G = 0.4$; $H_t = 0.6$;

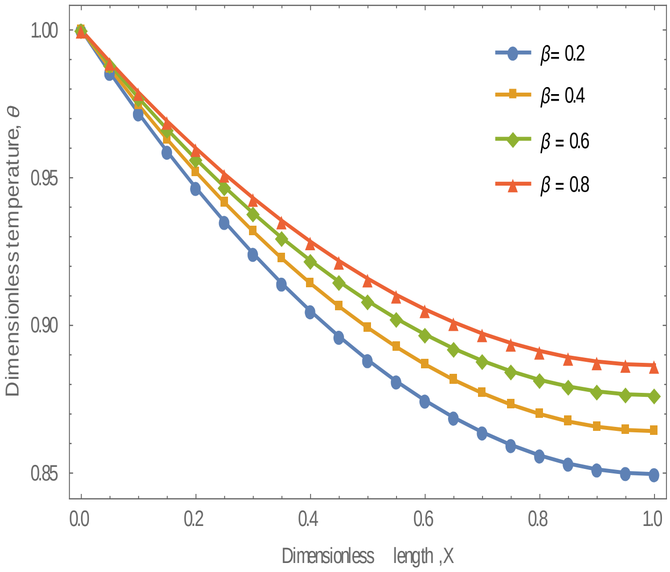

Figure 7. Effect of thermal conductivity parameter on the adimensional temperature when $S_p = 0.5; \mathrm{Pe} = 0.5; G = 0.4; H_t = 0.6; M = 0.3;$

Figure 8 and 9 show the impact of internal heat generation parameters on the adimensional temperature distribution in the fin. The figure shows that as internal heat generation parameters, *H* and *G* increase, the adimensional temperature in the fin increases. This is because as the generated internal heat in the fin increases, the conductive heat transfer in the fin increases and this in consequent increases the temperature of the extended surface.

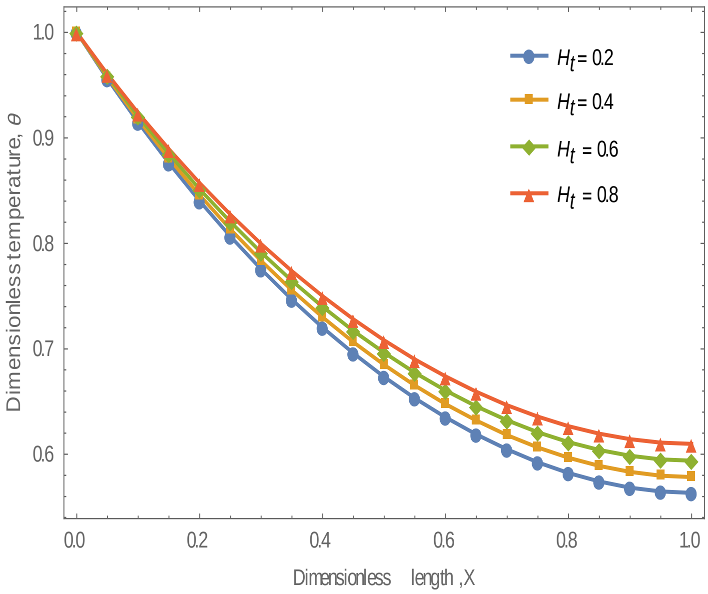

Figure 8. Effect of Internal heat generation parameter on the adimensional temperature when $\beta = 2; M = 2; S_p = 5; \mathrm{Pe} = 2; G = 0.4;$

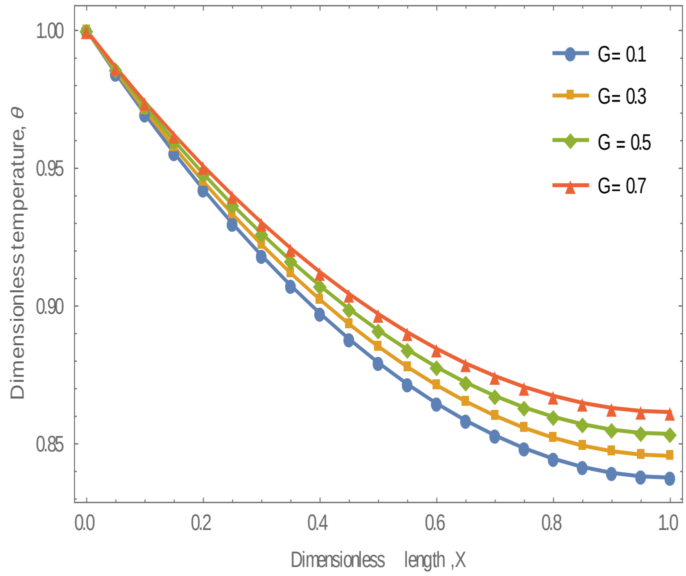

Figure 9. Effect of Internal heat generation parameter on the adimensional temperature when $\beta = 0.2; M = 0.3; S_p = 0.5; \mathrm{Pe} = 0.5; H_t = 0.6;$

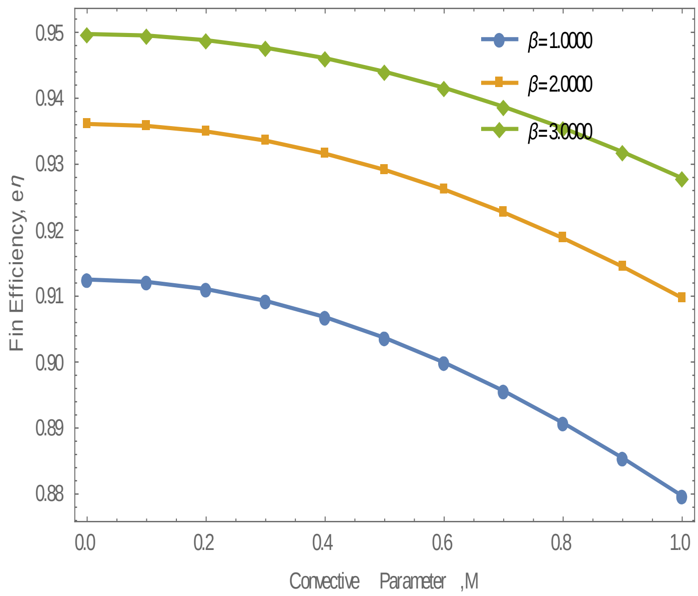

Figure 10. Effect of convective and thermal conductivity parameters on the efficiency when $H_t = 0.6; S_p = 0.7; G = 0.4; \mathrm{Pe} = 0.5;$

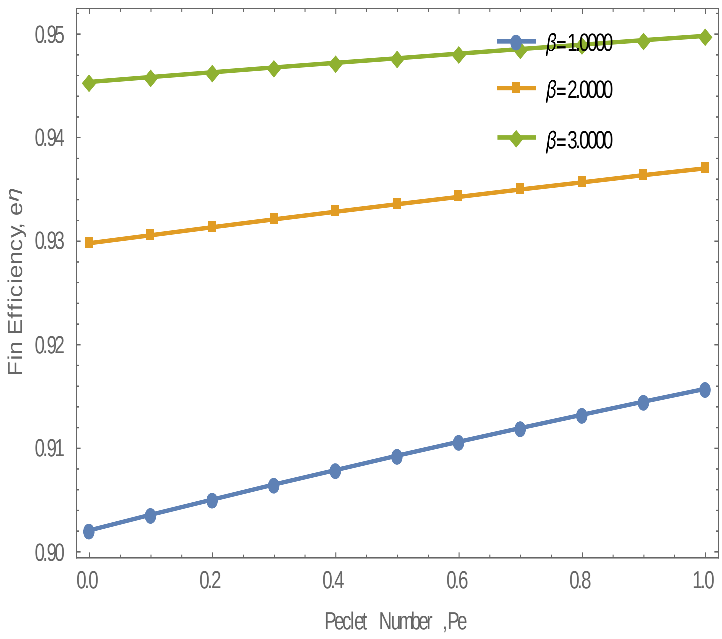

Figure 11. Effect of Peclet number and thermal conductivity parameters on the efficiency when $H_t = 0.6; M = 0.3; S_p = 0.7; G = 0.4;$

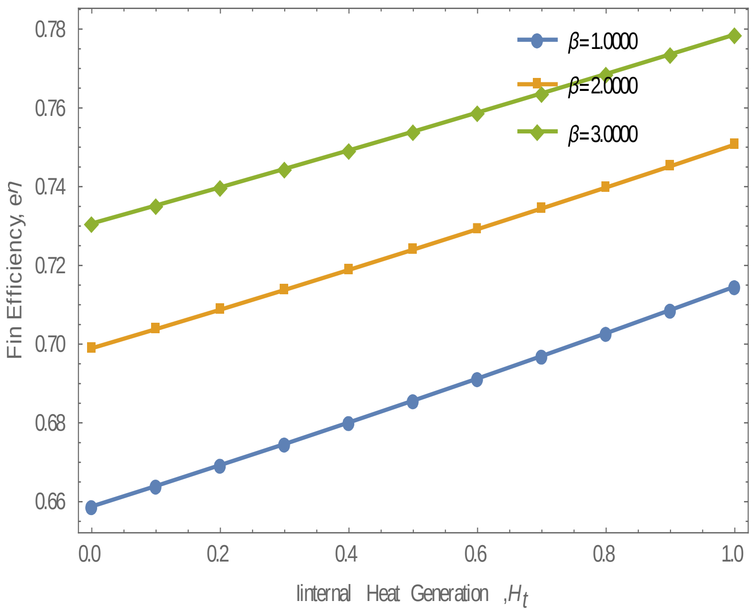

Figure 12. Effect of thermal conductivity and internal heat generation parameters on the efficiency when $M = 2; S_p = 5; G = 0.4; \mathrm{Pe} = 2;$

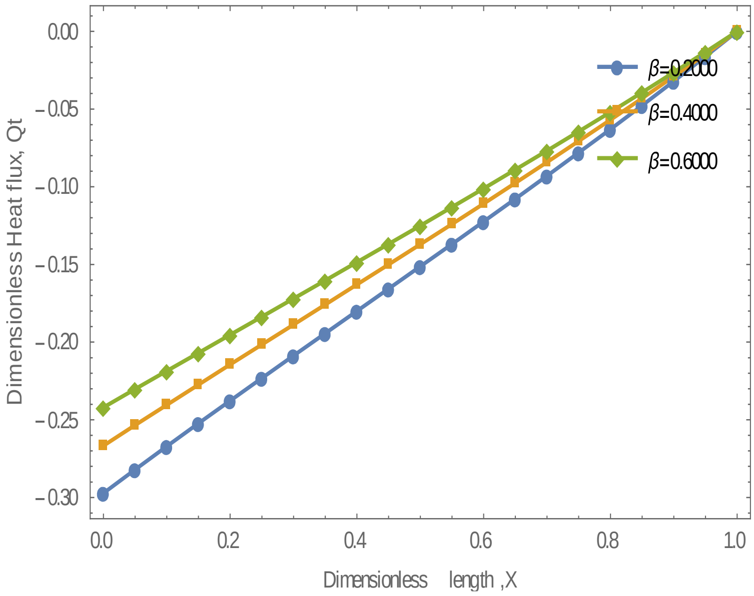

Figure 13. Effect of thermal conductivity parameter on the heat flux across the fin length when $H_t = 0.6; M = 0.3; S_p = 0.5; G = 0.4;$ Pe $= 0.5;$

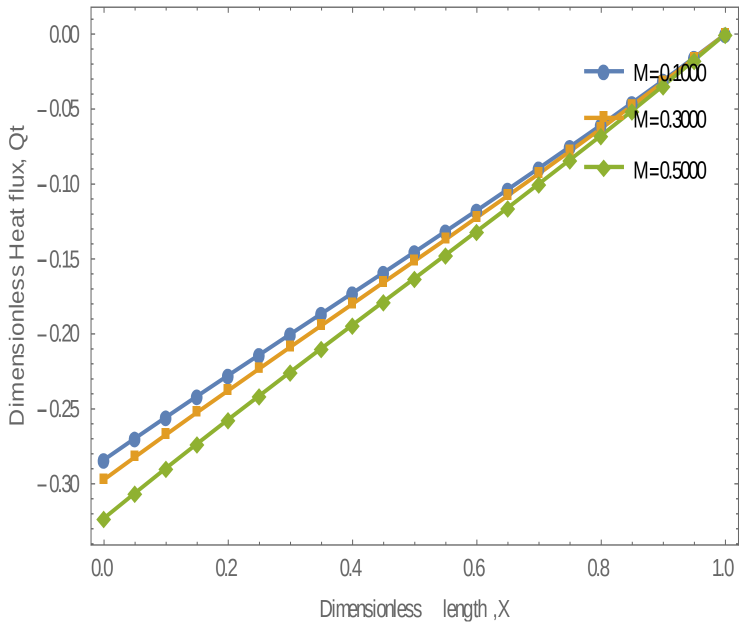

Figure 14. Effect of convective parameter on the heat flux across the fin length when $\beta = 0.2; H_t = 0.6; 0.5; G = 0.4;$ Pe $= 0.5.$

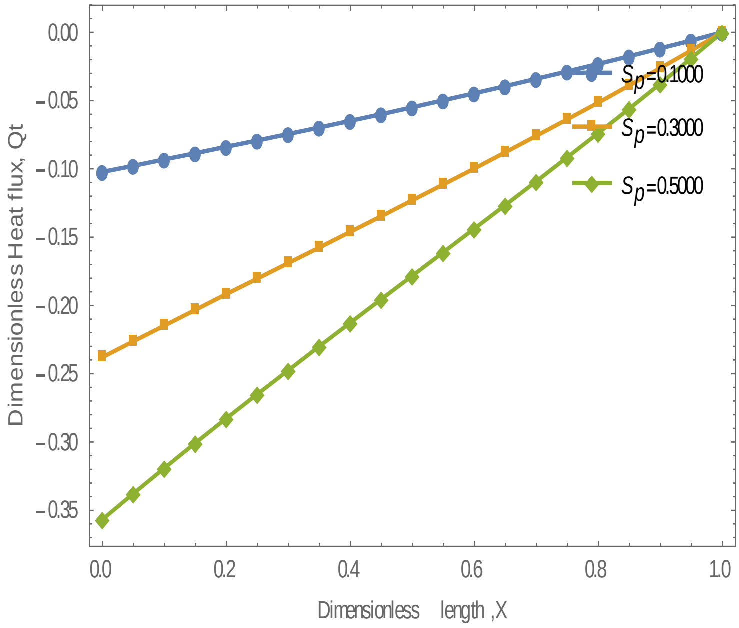

Figure15. Effect of porous parameter on the heat flux across the fin length when $\beta = 0.2; H_t = 0.6; 0.5; G = 0.4; \mathrm{Pe} = 0.5;$

Figures 10-12 show the effects of Peclet number, Hartmann number, porous and convective parameters on the thermal efficiency of the moving fin. It is shown that the fin efficiency increases as the thermal conductivity increases while the fin efficiency decreases as the convective parameter (and the porous parameter) increases as shown in Figure 10. It should be stated that the efficiency of the fin approaches a value of unity as the thermal conductivity parameter increases.

The fin efficiency increases as the Peclet number increases as shown in Figure 11. Such increase in the Peclet number is an indication of increase in the fin speed which augments the rate of heat transfer in the fin and enhances the fin efficiency.

The impact of internal heat generation on the fin thermal efficiency is shown in Figure 12. It is depicted in the that the fin efficiency increases with increase in the internal heat generation.

Generally, heat flux increases across the fin length. Figure 13-15 display the temperature gradient across the length of the fin for the varying value of the fin thermal conductivity, convective and porosity parameters. The figure shows that the fin temperature gradient numerically decreases as the thermal conductivity parameter increases. However, the fin temperature gradient numerically increases as the porosity and convective parameter increases.

7. Conclusion

In this paper, finite element method has been successfully employed in the thermal analysis of a moving convective porous fin with temperature dependent thermal conductivity and internal heat generation. Some interesting results were obtained through the parametric and sensitivity analyses. The results showed that when the convective and porous parameters increase, the adimensional fin temperature decreases. However, the value of the fin temperature is amplified as the value Peclet number is enlarged. Also, an increase in the thermal conductivity and the internal heat generation cause the fin temperature to fall and the rate of heat transfer from the fin to decrease. Therefore, the operational parameters of the fin must be carefully selected to avoid thermal instability in the fin.

Nomenclature

A Fin cross sectional area
h Coefficient of heat transfer.
C_p Fin specific heat capacity.
Da Darcy number.
g Gravity constant.

h Coefficient of heat transfer
k Fin thermal conductivity.
k_{eff} Porous fin effective thermal conductivity.
K Porous fin Permeability.
L Length of the fin.
M Dimensionless Convective parameter.
$\dot{m}$ fluid mass flow rate.
P Fin perimeter.
Pe Peclet number.
Q Adimensional heat transfer rate per unit area.
q internal heat generation
q_T Heat flux.
Sp Porosity parameter.
δ Fin thickness
T Fin temperature.
T_∞ Ambient temperature.
T_b Fin base temperature.
V_w Fin velocity.
w Fin width.
x Fin axial length.
X Adimensional axial length of the fin.
Ht Adimensional internal heat generation parameter
G Heat Generation number

Greek Symbols

β fin Thermal conductivity parameter.
θ Adimensional temperature.
η Fin efficiency.
υ Kinematic viscosity.
ρ Density of the fluid.
λ Measure of thermal conductivity variation with temperature.

Subscripts

s Solid properties.
f Fluid properties.
eff Effective porous properties.

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